Recovering Stars in Macroeconomics*

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Abstract

Many key macroeconomic variables such as the NAIRU, potential GDP, and the neutral real rate of interest — which are needed for policy analysis — are latent. Collectively, these latent variables are known as *'stars'* and are typically estimated using the Kalman filter or smoother from models that can be expressed in State Space form. When these models contain more shocks than observed variables, they are *'short'*, and potentially create issues in recovering the star variable of interest from the observed data. Recovery issues can occur when the model is correctly specified and all its parameters are known. In this paper, we summarize the literature on shock recovery and demonstrate its implications for estimating stars in a number of widely used models in policy analysis. The ability of popular and recent models to recover stars is shown to vary considerably. We suggest ways this can be addressed.

Keywords: Kalman filter and smoother, State Space models, shock recovery, short systems, natural rate of interest, macroeconomic policy, Beveridge-Nelson decomposition.
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1 Introduction

Stars were historically used as a navigational tool to guide travellers on a journey. Today, stars play a similar role in the conduct of macroeconomic policy. When an asterisk is attached to variables such as output, interest rates, or inflation, these variables are collectively known as *'stars'*, and the reference is to an equilibrium state towards which the economy is expected to adjust. Potential output, the neutral real rate of interest and the Non-Accelerating Inflation Rate of Unemployment (NAIRU) are prominent stars.

Stars might be thought of as the steady-state values that exist in some theoretical model. Since they would be functions of the model's parameters, any changes in the star variable itself would necessitate changes in the parameters of the model. Often, such changes are difficult to account for, because stars are likely to be complex, non-linear functions of the model's parameters. Consequently, a simpler, commonly used, alternative treats the star variable as a latent exogenous process, with a popular choice being a driftless random walk.

As the star variable is not directly observed, it needs to be estimated from data. This is typically done with a State Space model that captures the latent variable and either a Kalman filter or smoother is used to extract a measure of the latent star variable. As will be seen, in almost all such implementations the number of shocks in the model exceeds the number of observed variables. Adopting the description of Forni *et al.* (2019), such a system is said to be '*short*'. A key finding from recent theoretical work on shock recovery is that it is *never* possible to recover *all* the shocks from a short system, so this raises the question of whether one can recover the star variable of interest.

The objectives of this paper are two-fold. First, we demonstrate the importance of the literature on shock recovery for the modelling of star variables and, more broadly, for policy analysis. While it is well known that the model parameters used to construct the stars are typically estimated imprecisely (e.g., Staiger *et al.*, 1997 and Laubach and Williams, 2003), particularly when this is done in real time (Orphanides and van Norden, 2002) and such an outcome creates practical issues for policy analysis, it is *not* well known that these models may be unable to recover the star variable of interest to the policy maker *even when the model is correctly specified and its true parameters are known*. Second, we provide a critical review of widely used models that were developed to provide estimates of stars for policy analysis.

Short systems, which *cannot recover all the shocks* in the model, are typically used to estimate stars. Nevertheless, *they may be able to recover the shocks that drive the star.* Moreover, *the ability to recover the star will vary from model to model.* Not all short systems are equal. Quantifying shock and star recovery is important information for researchers. Understanding a model's limitations regarding recovery can prompt further investigation of variants or alternatives to it. It is meant to complement diagnostics which focus on model misspecification, but it examines a very different issue. Likewise it provides different information to confidence or highest posterior density intervals of a star - for example, recovery is about the model's properties in ideal situations. We believe it is important for a policymaker, who may need to choose between different star estimates from various alternative models, to know each model's ability to recover the star.

Star recovery is *not* a parameter estimation issue. In all our analysis, we evaluate star recovery by assuming that the model being used is correctly specified and that its true parameters are known. The focus is on whether the model used to find the star can do that, even when there are no issues about its ability to fit the data. To us, the ability of a model to recover the star variable of interest that it was designed for is a minimal, desirable property that needs to be satisfied. This is not to deny the fact that, even if we find that the model under investigation can recover the star, there are other dimensions, such as the real-time reliability of its estimates, which are also desirable and highly relevant for the conduct of policy. Recovery essentially measures whether the assumed system contains enough information to find the shocks or stars of interest when its parameters are known with certainty.

Recovery performance is not binary — different models will give different degrees of recovery. It is thus important to have a standardized way to measure and communicate a model's ability to recover a star. We demonstrate how this can be done simply with a correlation coefficient. This correlation coefficient will help policymakers discern between star estimates from multiple models.

It is our view that those presenting estimates of stars from short systems should be obliged to show that the model *can* in fact recover the star variable of interest. The correlation coefficient that we propose should be routinely reported when estimating star variables, in the same way that one reports standard errors or confidence intervals to gauge the level of uncertainty surrounding point estimates of parameters computed from a statistical model. Presently, this is not being done.

The remainder of the paper is structured as follows. Section 2 defines the concept of recoverability, the implications of a short system for recoverability, and distinguishes between recoverability and statistical uncertainty. It also provides a summary of recently developed approaches to assess shock recovery and explains how these can be extended to examine recoverability of star variables.

In Section 3, several applications aimed at recovering star variables are provided. There are many star variables; to focus our analysis we primarily examine models intended to estimate the neutral real rate of interest. We analyze whether the neutral rate can be recovered by the influential Laubach and Williams (2003) model and its later updates in Holston *et al.* (2017, 2023). An extension of that model in McCririck and Rees (2017) is examined. This variant is used in the Reserve Bank of Australia's policy model (Ballantyne *et al.*, 2020). In all these models, we find that the ability to recover the star variables of interest is limited. This lack of recovery persists in an extended version which includes an interest rate rule.

Section 4 turns to recent alternative approaches. Schmitt-Grohé and Uribe (2022) propose a different type of structural model that incorporates permanent monetary shocks to estimate stars. Their approach is more successful in recovering the neutral real rate. However, it crucially depends on the magnitude of one parameter, which attributes what might be thought to be an unrealistically high percentage (nearly 80%) of the variation in output growth to the neutral real rate shock. Once this parameter is set to what seems a more reasonable value, the natural real rate cannot be recovered.

This section also investigates an entirely different approach that avoids providing an explicit structural model for the star variable, instead defining it via a Beveridge-Nelson (1981) decomposition. Two such approaches, namely, Morley, Tran and Wong (2023), and Lubik and Matthes (2015), are examined. The former is more successful than the Laubach and Williams (2003) based approaches, although it also does not completely recover the star.

In Section 5 we discuss stochastic volatility. This is an increasingly popular feature of macroeconomic models that may unintentionally obscure the stars since it results in short systems. Lastly, Section 6 describes an alternative approach to estimating stars which has been used by Okimoto (2019) to measure trend inflation. Okimoto does not add extra shocks to the system and thereby avoids recovery problems associated with short systems. Section 7 concludes the paper.

2 Recovering Latent Variables from Models

2.1 What is recoverability?

To answer this question, we utilize models that can be written in the following State Space Form (SSF):

$$\text{Measurement}: \quad Z_t = D_1 X_t + D_2 X_{t-1} + R\varepsilon_t \tag{1}$$

$$\text{State}: \quad X_t = AX_{t-1} + Q\varepsilon_t, \tag{2}$$

where ε_t is a multivariate normal distributed random variable with a zero mean and identity covariance matrix, D_1, D_2, A, R are Q are conformable system matrices, Z_t contains the observed variables and X_t the latent states.[1] There may be identification and other econometric issues in estimating D_1, D_2, A, R and Q when there are more shocks than observables. Such issues are discussed in Buncic (2024) for the model of Holston, Laubach and Williams (2017, HLW) that aims to capture the neutral real rate of interest.[2] Despite the empirical importance of such estimation problems, we will assume that the numerical values provided in the papers of the models considered are the true values. This is done to abstract from estimation issues in our analysis, since our concern is the recovery of shocks and the star variable from a given model and parameters.

There are two ways of looking at equations (1) and (2) describing the relationship between observed variables, latent states and shocks. One of these makes assumptions about the *assumed* shocks ε_t and, given D_1, D_2, A, R, Q, characteristics such as variances and covariances of the random variable Z_t can be determined. In this form, the analysis is working from the right to left of the SSF equations. In addition to knowing the model parameters there are *auxiliary assumptions* about the nature of ε_t, for instance, that they are uncorrelated. Given these, the SSF can be used to tell the investigator about the *assumed model properties of* variable Z_t, e.g., what the assumed model says about the variance of Z_t.

A different perspective comes from introducing data into the model. Now the LHS of (1) has the data Z_t^D, and this is used to recover the shocks. The data will be used to produce either filtered or smoothed shocks. We will largely work with *smoothed shocks* and denote them by $E_T\varepsilon_t$. Thus, smoothed shocks at time t are defined as the expectation of the shock ε_t using *all the* T observations in the sample. Filtered shocks are denoted by $E_t\varepsilon_t$, and are estimated using data up to time period t. Designating the data as Z_t^D, the (Kalman smoothed)

[1]To reduce the size of the state vector, we will utilize the SSF and the Kalman Filter of Nimark (2015), which adds a lagged state to the measurement equation, and the smoother of Kurz (2018).

[2]This is sometimes also called the natural real rate; we use these terms interchangeably.

system can be expressed as:

$$Z_t^D = D_1 E_T X_t + D_2 E_T X_{t-1} + R E_T \varepsilon_t \quad (3)$$

$$E_T X_t = A E_T X_{t-1} + Q E_T \varepsilon_t. \quad (4)$$

Given a set of data (and D_1, D_2, A, R, Q), one obtains smoothed shocks from the Kalman smoother. *Recovery* is achieved when we can obtain ε_t from the data using $E_T \varepsilon_t$. *If it is possible to recover ε_t, then it is possible to recover the latent variables X_t, as these are a function of the shocks.* Recovering shocks and stars is intrinsically interrelated.

It is important to highlight here once more that recovery is *not* a parameter estimation issue, as we have assumed all parameters to be known. It is an *information* issue. It examines whether it is possible to recover the assumed (theoretical) shocks from the available information — the data — when using the estimate $E_T \varepsilon_t$ and the known parameters.

The ability of a model to recover the latent star variable one is trying to estimate when the model is correct and all its parameters are known would seem to be a self-evident, minimal property that any model should satisfy, especially when used to make policy decisions.

2.2 Implications of short systems for recoverability

When the number of shocks equals the number of observed variables, then ε_t and $E_T \varepsilon_t$ generally coincide and recovery is satisfied. Therefore, whether the shocks ε_t have their assumed properties can be directly assessed using the estimated $E_T \varepsilon_t$. Conversely, when there are more shocks than observables, the system is said to be *'short'* (Forni *et al.*, 2019), and recovery is not ensured.

To illustrate the implications of this, it is useful to think about a *'short'* system in the simplest possible scenario where we have one observed variable and two shocks ε_{1t} and ε_{2t} that are $n.i.d(0, 1)$ and uncorrelated with each other. We then obtain the following two equations corresponding to (1) and (3):

$$Z_t = \varepsilon_{1t} + \varepsilon_{2t} \tag{5}$$

$$Z_t^D = E_T\varepsilon_{1t} + E_T\varepsilon_{2t} \tag{6}$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} E_T\varepsilon_{1t} \\ E_T\varepsilon_{2t} \end{bmatrix}$$

$$= GE_T\varepsilon_t. \tag{7}$$

From the relation in (7), it is apparent that $E_T\varepsilon_t$ cannot be recovered uniquely from Z_t^D, because G is not a square matrix, and thus does not have an inverse. If G was square, then we would have a solution for $E_T\varepsilon_t = G^{-1}Z_t^D$, and G^{-1} would satisfy $GG^{-1}G = G$. When G is not square, G^{-1} is replaced by a generalized inverse G^+ that satisfies $GG^+G = G$. Then $E_T\varepsilon_t = G^+Z_t^D$.

Letting $G^+ = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$, $GG^+G = G$ implies that

$$g_1 + g_2 = 1,$$

and so there are many values for g_1. To select one, it is common to find the value that minimizes $G^{+\prime}G^+$. For this case it yields

$$G^+ = \begin{bmatrix} .5 \\ .5 \end{bmatrix},$$

which implies that $E_T\varepsilon_{1t} = .5Z_t^D = E_T\varepsilon_{2t}$. That is, the smoothed shocks $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{2t}$ are *identical* to one another, and thus cannot be separated using the data.[3]

A different way of looking at this is to observe that the Kalman smoother would ask what value of g_1 would make $E_T\varepsilon_{1t} = g_1 Z_t$ as close as possible to ε_{1t} in a mean squared

[3] The Kalman filter and smoother effectively work with a generalized inverse.

error (MSE) sense. That means minimizing $E[(\varepsilon_{1t} - g_1 Z_t)^2]$ with respect to g_1 and so

$$\begin{aligned} & \min_{g_1} E[(\varepsilon_{1t} - g_1\varepsilon_{1t} - g_1\varepsilon_{2t})^2] \\ = \quad & \min_{g_1}[(1 - 2g_1 + 2g_1^2] \\ \Longrightarrow \quad & g_1 = .5. \end{aligned}$$

Consequently $\text{Var}(\varepsilon_{1t} - .5Z_t) = .5$.

The example is useful for illustrating some of the points made above. First, if one only knows that $\text{Var}(\varepsilon_{2t}) = 1$, it will be necessary to estimate $\text{Var}(\varepsilon_{1t})$ from data to produce estimates of G and the smoothed shock $E_T\varepsilon_{1t}$. As explained earlier we want to avoid this extra complication so as to ask whether the model can recover the shocks in the best possible circumstances. Second, one can compute a decomposition of the $\text{Var}(Z_t)$ using just the assumed model and the auxiliary assumptions about the shocks in it. This would say that each shock contributes 50% of the $\text{Var}(Z_t)$. Finally, the variance of the data $\text{Var}(Z_t^D)$ cannot be decomposed in such a way. As seen in (6), a decomposition of the data involves smoothed shocks, but these are perfectly correlated with each other, so the smoothed shock explains 100% of the variance of Z_t — there is no decomposition into the effects of separate shocks. If, for example, one thought of Z_t as inflation and ε_{1t} and ε_{2t} as demand and supply shocks, we would find that either "demand" or "supply" explains *all* of the variation in inflation.

The general lessons learned from the above simple example are that a variance decomposition of Z_t with respect to the ε_t — as is suggested in Plagborg-Møller and Wolf (2022) — comes from the model and its auxiliary assumptions. In a short system this is *not a variance decomposition of the data* since some of the estimated shocks will be correlated and this is counter to the central assumption underlying a variance or variable decomposition that the smoothed shocks are uncorrelated (Pagan and Robinson, 2022). Indeed, in more complex models the relationship between smoothed shocks is not just a simple correlation, but a complex dynamic one, making it very difficult to assess the economic importance of changes in the star variables. The models we examine below have such complex relationships.

2.3 Assessing shock recoverability

In a short system, not *all* of the assumed shocks can be recovered. However, it may be possible to recover *some* shocks from the model being used, potentially those of relevance to policy makers which determine the star variable of interest. Consequently, we need to review methods in the literature in order to show which shocks can be recovered. We discuss how these methods are related.

Forni *et al.* (2019), building on Sims and Zha (2006), developed a deficiency index to determine whether in an SVAR there is sufficient information to recover a particular shock from current and past information. They find that it may be possible to do so, even when the system as a whole is not invertible. Their deficiency index fundamentally examines the recovery of the shock from its filtered estimates.

Chahrour and Jurado (2022) extended the concept of invertibility of an SVAR to consider an expanded information set which incorporated *future* information. They termed this *recoverability* and looked at $\phi = \text{Var}\left(\varepsilon_{1t} - E_T\varepsilon_{1t}\right)$ to assess that. By the definition of a conditional expectation, $\varepsilon_{1t} = E_T\varepsilon_{1t} + v_t$, where the signal $E_T\varepsilon_{1t}$ is uncorrelated with the "noise" v_t. Hence, $\text{Var}\left(\varepsilon_{1t}\right) = \text{Var}\left(E_T\varepsilon_{1t}\right) + \text{Var}\left(v_t\right)$ and $\phi = \text{Var}\left(\varepsilon_{1t} - E_T\varepsilon_{1t}\right) = \text{Var}\left(v_t\right)$. Because $\text{Var}\left(\varepsilon_{1t}\right) = 1$, we have $0 \leq \phi \leq 1$.

Pagan and Robinson (2022) noted that the Kalman smoother gave $E_T\varepsilon_{1t}$, as well as ϕ, the latter via the MSE of the state vector, defined as $E\left[\left(X_t - E_TX_t\right)\left(X_t - E_TX_t\right)'\right]$ and commonly denoted by $P_{t|T}$. To compute ϕ therefore all that needs to be done is to add the shock to the state vector of the SSF and apply the Kalman smoother. This is done with the *steady-state* version of the MSE (and Kalman filter), and we will call this $P^*_{t|T}$. As it is the steady-state version recovery is assessed as if there is an infinite amount of future data. This approach can be implemented for a wide range of models.

Plagborg-Møller and Wolf (2022) alternatively suggested the use of the R^2 from a population regression of the model shock ε_{1t} against the smoothed shock $E_T\varepsilon_{1t}$.[4] As $E_T\varepsilon_{1t}$ is the "signal" and Var $(\varepsilon_{1t}) = 1$, it follows that this $R^2 = \text{Var}(E_T\varepsilon_{1t})$, and $\text{Var}(v_t) = \phi = 1 - R^2$. An appealing aspect of these measures is that they generalize to non-linear models, provided a filter and smoother to compute $E_T\varepsilon_{1t}$ is available. Working with $\rho = \sqrt{R^2}$ produces the correlation between ε_{1t} and $E_T\varepsilon_{1t}$. That may be a more meaningful measure for decision makers to interpret. In any case, $\phi = 1 - \rho^2$ so it is a simple matter to convert one measure into the other.[5]

When the shock is recoverable, $E_T\varepsilon_{1t} = \varepsilon_{1t}$ and $\phi = 0$ $(\rho = 1)$. If it fails to be recoverable $\phi = 1$ $(\rho = 0)$. In the latter case,Var $(E_T\varepsilon_{1t}) = 0$ and this is as far away from $\text{Var}(\varepsilon_{1t}) = 1$ as one can get.

In summary, these measures of recoverability all are essentially answering the question:

> If a set of the observed series of infinite length were generated from the model with known parameters, could we then recover the shocks from these data?

2.4 Assessing star recoverability

The primary message of this paper is that if a star variable is modelled as a function of unrecoverable shocks, then the star variable *itself* cannot be recovered from the data. Therefore, recovery can be assessed by the measures above, such as ϕ or ρ, for the relevant shocks. Nonetheless, there are three aspects that require attention. First, as the star may be a combi-

[4] Instead of calculating this quantity from the smoothed shocks obtained from the data, it is more appropriate to use its population counterpart, which can be found by simulating a long sequence of data from the model, applying the Kalman filter and smoother to obtain $E_T\varepsilon_{1t}$, and then computing the R^2 using the known ε_{1t} and $E_T\varepsilon_{1t}$.

[5] Because we generally use the steady state filter to find P^* there can be some minor differences between the computed ϕ and $1 - \rho^2$, so we will present both P^* and ρ.

nation of multiple shocks, focusing directly on recovery of the star, rather than the shocks, can be useful. Second, in many instances, it is assumed that the star variable follows a non-stationary process. In this case, it would be the correlation between the appropriately differenced series and its smoothed counterpart that is assessed for recovery. Third, it is useful to normalize the star variable by its standard deviation, which is similar to working with standardized shocks that have a unit variance by definition.

The correlation is a highly useful and intuitive way of communicating the degree to which a model can recover a star variable. Indeed, our view is that one should always report such a correlation measure in the same way that confidence intervals are routinely reported to gauge the level of statistical uncertainty surrounding point estimates of parameters. It can be used in at least two ways. First, as a guide to a policymaker. The correlation is essentially an indication of whether the model is informative about the star variable under the most ideal circumstances — when the model is correctly specified and its parameters are known. Often policy institutions maintain a "suite of models". There are numerous ways to estimate stars, and a policymaker may down-weight those with a low correlation. Indeed, one could consider "model averaging" of star estimates using the normalized correlations as aggregation weights.[6]

The second way that the correlation can be used is by researchers. A low correlation can prompt a re-examination of the model. In some cases, like the Ireland (2004) model examined in Pagan and Robinson (2022), an extra observed variable may be readily available, so that the system can be augmented to no longer be short. However, when estimating stars this may be difficult to achieve, and sometimes working with a short system may be unavoidable. Still, it may be the case that the model is short because of the introduction of

[6]Of course this brings up the issue of how low a value of ρ would be acceptable to someone using a model to compute a star variable. It is the same issue as choosing a critical value for a $t-$ratio. The costs of making a bad decision because ρ is low need to be stated and will depend on the decision maker. Our point is that if one is using a model to measure a star, a decision maker (and a reader) would want to know the magnitude of ρ.

shocks that are simply added on for convenience, e.g., to capture measurement errors in the data. These could be removed to produce a system that is not short.

A subtlety with using the correlation to inform model design is that it is possible for a correctly specified model to have a low correlation. Studying recoverability in a star model is assessing its ability to deliver on its intended aim in the best possible circumstances. Whether a model is correctly specified should still be assessed with standard model diagnostics. Implementing some of these tests is less straightforward in short systems as its smoothed shocks must be correlated.[7]

Introducing more information by adding observables to a short system with a low correlation is always desirable. One possibility is to expand the set of observables with forecasts.[8] Adding forecasts and relating them to model-consistent forecasts in the estimation of potential output was implemented by Alici *et al.* (2017) with the motivation of lessening endpoint issues. Although the system remains short, recovery could conceivably be improved. Forecasts can also be used as a measure of conditional expectations, for example, inflation expectations in a Phillips curve to estimate the NAIRU.[9] A recent noteworthy example is Crump *et al.* (2019), who include survey forecasts from professional forecasters.[10] In our opinion, exploring further how forecasting data sets can be used to aid the recovery of stars seems to be a productive area for future research.

[7] Pagan and Robinson (2022) present an indirect inference approach for assessing if the correlation in the estimated shocks aligns with what was assumed by the model.

[8] See, for instance, Hirose and Kurozumi (2021) in the news literature, examined in the working paper version of this paper.

[9] The importance of inflation expectations measures for NAIRU estimates is discussed by Ellis (2019). Forecasts have been used for conditional expectations in news models, e.g. Barsky and Sims (2012).

[10] Crump *et al.* (2019) allow for measurement error, which is sensible as the expectations of households and professional forecasters can differ — see Dräger *et al.* (2016).

3 Applications Aimed at Recovering Stars

To illustrate recoverability issues, we first consider the model used by Laubach and Williams (2003) to estimate the neutral real rate and their subsequent updates, Holston *et al.* (2017) and (2023). We then turn to a model that builds upon Laubach and Williams (2003), namely McCririck and Rees (2017), and estimates three stars simultaneously: the NAIRU, the neutral real rate, and potential output. In all of these cases, there are problems with recovering the star variables, particularly the neutral real rate.

We provide code and detailed documentation to replicate the results from models in this paper on a GitHub repository at: `https://github.com/4db83/Recovery-code`. The repository also includes additional models, such as the Hodrick and Prescott (1997) filter.

3.1 Recovering the Neutral Real Rate - Laubach and Williams (2003)

One of the most influential models of the neutral real rate r_t^* of the past two decades is that of Laubach and Williams (2003, LW). There exist numerous alternative and/or extended versions of the LW model in the literature, and these are widely used at central banks and other policy institutions. The LW model consists of the following equations:

$$\tilde{y}_t = \alpha_1 \tilde{y}_{t-1} + \alpha_2 \tilde{y}_{t-2} + \frac{a_r}{2} \sum_{i=1}^{2} (r_{t-i} - r_{t-i}^*) + \sigma_1 \varepsilon_{1t} \tag{8a}$$

$$\pi_t = B(L)\pi_{t-1} + b_I(\pi_t^I - \pi_t) + b_o(\pi_{t-1}^o - \pi_{t-1}) + b_y \tilde{y}_{t-1} + \sigma_2 \varepsilon_{2t} \tag{8b}$$

$$\Delta z_t = \sigma_3 \varepsilon_{3t} \tag{8c}$$

$$\Delta y_t^* = g_{t-1} + \sigma_4 \varepsilon_{4t} \tag{8d}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{8e}$$

$$r_t^* = c4 g_t + z_t, \tag{8f}$$

where $\tilde{y}_t = (y_t - y_t^*)$ is the output gap, y_t is (100 times the log of real) GDP, y_t^* is potential GDP, r_t is a real interest rate, r_t^* is the neutral real rate, π_t, π_t^I and π_t^o are various measures of inflation, and $B(L)$ is a lag polynomial to capture the dynamics in inflation. There are evolving processes for the trend growth of GDP g_t, and '*other determinants*' z_t, which affect r_t^*. There are a total of five shocks $\eta_{it} = \sigma_i \varepsilon_{it}, \forall i = 1, \ldots, 5$, with standard deviations $\{\sigma_i\}_{i=1}^5$, and the error terms $\{\varepsilon_{it}\}_{i=1}^5$ have unit variances as before. All relevant model parameters are taken from LW and are documented in the GitHub repository associated with this paper. While the focus of LW is on estimating r_t^* defined in (8f), trend growth g_t is also estimated.

In order to assess recoverability as outlined in Section 2.3, we write LW's model in (8) in shock recovery SSF so that all observables are contained in Z_t on the LHS of (1), and all shocks and remaining latent states are collected in the state vector X_t. Doing this yields the measurement equations:

$$Z_{1t} = y_t^* - \alpha_1 y_{t-1}^* - \alpha_2 y_{t-2}^* + \frac{a_r}{2} \sum_{i=1}^{2} r_{t-i}^* + \sigma_1 \varepsilon_{1t}, \tag{9}$$

$$Z_{2t} = b_y y_{t-1}^* + \sigma_2 \varepsilon_{2t}, \tag{10}$$

and the relevant state dynamics are given by:

$$\Delta y_t^* = g_{t-1} + \sigma_4 \varepsilon_{4t} \tag{11}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{12}$$

$$\Delta r_t^* = c4 \sigma_5 \varepsilon_{5t} + \sigma_3 \varepsilon_{3t}. \tag{13}$$

The state vector of the shock recovery SSF consists of:

$$X_t = \begin{bmatrix} y_t^* & y_{t-1}^* & g_t & r_t^* & r_{t-1}^* & \varepsilon_{1t} & \varepsilon_{2t} & \varepsilon_{3t} & \varepsilon_{4t} & \varepsilon_{5t} \end{bmatrix}'. \tag{14}$$

The LHS observable part of Z_t is given by:

$$y_t - \alpha_1 y_{t-1} - \alpha_2 y_{t-2} + \frac{a_r}{2} \sum_{i=1}^{2} r_{t-i} = Z_{1t} \tag{15}$$

$$\pi_t - B(L)\pi_{t-1} - b_I(\pi_t^I - \pi_t) - b_o(\pi_{t-1}^o - \pi_{t-1}) - b_y y_{t-1} = Z_{2t}. \tag{16}$$

The full SSF corresponding to (1) and (2) for LW's model can thus be written as:

$$\begin{aligned}
\underbrace{\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix}}_{Z_t} &= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & -b_3 & 0 & 0 & 0 & 0 & \sigma_2 & 0 & 0 & 0 \end{bmatrix}}_{D_1} X_t \\
&+ \underbrace{\begin{bmatrix} -a_1 & -a_2 & 0 & -\frac{a_3}{2} & -\frac{a_3}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{D_2} X_{t-1} + \underbrace{\mathbf{0}_{2\times 5}}_{R} \underbrace{\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}}_{\varepsilon_t}
\end{aligned} \tag{17a}$$

$$\underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ r_{t-1}^* \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \varepsilon_{3t-1} \\ \varepsilon_{4t-1} \\ \varepsilon_{5t-1} \end{bmatrix}}_{X_{t-1}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \\ 0 & 0 & \sigma_3 & 0 & 4c\sigma_5 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}}_{\varepsilon_t} \tag{17b}$$

Note here that the SSF corresponding to LW's structural model has *five shocks*, but only *two observed variables*. This means that it will not be possible to recover more than two unique shocks from this model. These could be linear combinations of all five shocks in the model, rather than any particular two of the five shocks in LW's model.

Given the SSF, we can follow the same process as outlined in Section 2.3 to determine which shocks are likely to be recoverable and which ones are not. For LW's model, the following recovery measures corresponding to the five shocks $\{\varepsilon_{it}\}_{i=1}^{5}$ are obtained:

Table 1: Recovery measures: Laubach and Williams (2003) shocks

Shocks:	$\varepsilon_{1t}(\tilde{y}_t)$	$\varepsilon_{2t}(\pi_t)$	$\varepsilon_{3t}(z_t)$	$\varepsilon_{4t}(y_t^*)$	$\varepsilon_{5t}(g_t)$
$\text{diag}(P_{t\|T}^*)$	0.6952	0.0146	0.9749	0.3353	0.9800
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.5521	0.9927	0.1585	0.8153	0.1415

The cost push shock, ε_{2t}, is recoverable. However, neither the '*other determinants*' shock ε_{3t} nor the trend growth shock ε_{5t} can be recovered. Note from equation (13) that these two shocks define the neutral rate. Therefore, the neutral rate itself will not be recoverable from this model. In fact, the (population) correlation between the smoothed estimate of the change in the neutral real rate and its true value is only 0.1764.[11] This, we believe, is useful information for any policymaker considering to use this model, which is distinct from confidence or highest posterior density intervals of the neutral rate, for example, as it assesses how the model would perform in the best possible circumstances. Figure 1 shows a graphical representation of recoverability of each of the shocks and Δr_t^* in LW's model.

A direct consequence of the lack of recoverability in LW's model is that the smoothed shocks of the Δy_t^*, Δg_t and Δz_t equations given in (8d), (8e) and (8c) are related through an identity. That is, defining $\eta_{it} = \sigma_i \varepsilon_{it}$, this identity involves the smoothed estimates of the trend growth shock $\Delta E_T \eta_{5t}$, the '*other determinants*' shock $\Delta E_T \eta_{3t}$, and the trend shock $E_T \eta_{4t}$:

$$\Delta E_T \eta_{5t} = 0.0266 \Delta E_T \eta_{3t} - 0.0018 E_T \eta_{4t}. \tag{18}$$

[11]The correlation is found through simulation and application of the Kalman filter and smoother to the simulated data. Alternatively, one can expand the SSF in (17) to include Δr_t^* at the end of the state vector as an extra state variable and compute the same recovery measures as before. Since the variance of Δr_t^* is not going to be unity, $P_{t|T}^*$ needs to be normalized by $\text{Var}(\Delta r_t^*)$ for the $P_{t|T}^*$ to be comparable to the remaining entries. The normalized $P_{t|T}^*$ for Δr_t^* is 0.9733.

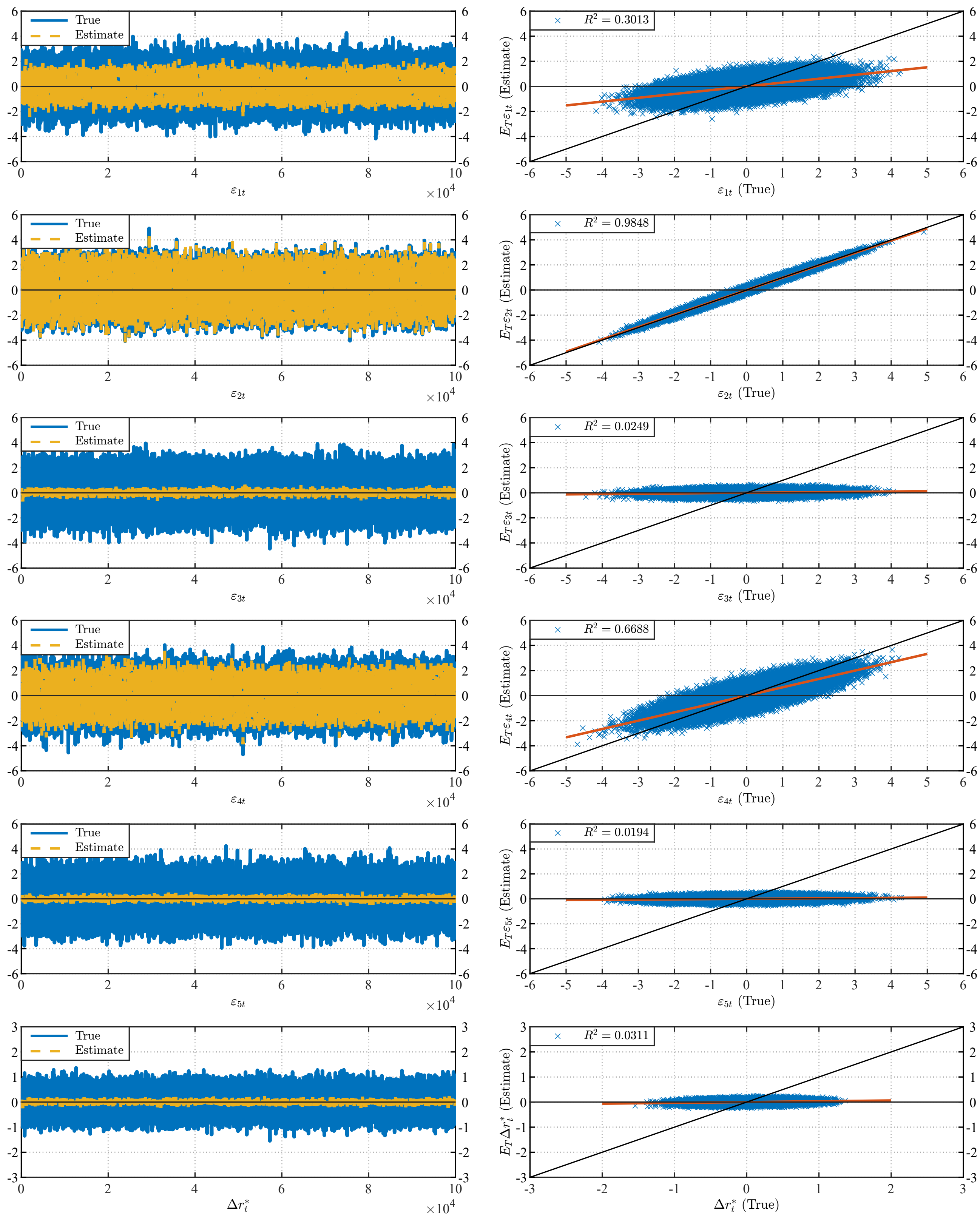

Figure 1: Comparison of the true shocks and change in the neutral real rate and their Kalman Smoothed estimates in Laubach and Williams (2003).

Moreover, from the smoothed states we can further establish the following two identities:[12]

$$E_T \Delta r_t^* = 4c E_T \eta_{5t} + E_T \eta_{3t}, \tag{19}$$

and

$$\begin{aligned} E_T \Delta r_t^* = {} & E_T \Delta r_{t-1}^* - 0.1388 E_T \eta_{1t} - 0.0008 E_T \eta_{2t} + 0.0196 E_T \eta_{4t} \\ & + 0.0665 E_T \eta_{1t-1} - 0.0272 E_T \eta_{4t-1}. \end{aligned} \tag{20}$$

These show that whatever $E_T \Delta r_t^*$ is measuring can be equally well explained by either (19) or (20). The latter involves a dynamic combination of smoothed demand, technology and Phillips curve shocks, while the former has smoothed values of the shocks meant to explain the neutral real rate. Consequently, the presence of a short system creates interpretation difficulties. LW's estimated model *cannot* distinguish which shocks are driving r_t^*.

Holston, Laubach and Williams (2017, HLW) provide an updated version of the original LW model using a somewhat different formulation of the Phillips curve equation (8b) estimated over a longer sample period, and one might ask whether recoverability in HLW improves over LW. Examining the recovery measures in Table 2 clearly this is not the case. The two shocks driving the natural rate again are not recoverable. Computing the correlation between the true model implied change and the estimated change in the natural rate ($E_T \Delta r_t^*$) yields a value of 0.1412 from HLW's model.

Table 2: Recovery measures: Holston, Laubach and Williams (2017) shocks

Shocks:	$\varepsilon_{1t}(\tilde{y}_t)$	$\varepsilon_{2t}(\pi_t)$	$\varepsilon_{3t}(z_t)$	$\varepsilon_{4t}(y_t^*)$	$\varepsilon_{5t}(g_t)$
$\text{diag}(P_{t\|T}^*)$	0.6979	0.0178	0.9913	0.3183	0.9746
$\text{Corr}(\varepsilon_t, E_T \varepsilon_t)$	0.5464	0.9907	0.0928	0.8280	0.1585

[12] η_{5t} has been multiplied by 4, reflecting that g_t is annualized.

Identities of the form in (18), (19), and (20) also exist for HLW's model, and once again provide two possible representations of what is driving changes in the neutral rate.[13]

Due to the impact of COVID-19 on the variables in HLW's model, Holston *et al.* (2023) modify the specification in HLW by allowing potential output to be impacted by government policy responses, which they measure using the Oxford policy tracker (Hale *et al.*, 2021), by allowing the variance of the shocks to temporarily increase (the approach is similar to Lenza and Primiceri, 2022). Since the modified model has an additional parameter κ that determines how large the step increase is in the variance, in Table 3 we provide recovery measures for two different κ values; a high value of $\kappa = 9.033$ and a low value of $\kappa = 1.676$, corresponding to the time periods 2020:$Q2 - Q4$ and 2022, respectively.

Table 3: Recovery measures: Holston, Laubach and Williams (2023) shocks

Shocks:	$\varepsilon_{1t}(\tilde{y}_t)$	$\varepsilon_{2t}(\pi_t)$	$\varepsilon_{3t}(z_t)$	$\varepsilon_{4t}(y_t^*)$	$\varepsilon_{5t}(g_t)$
			$\kappa = 9.033$		
$\text{diag}(P^*_{t\|T})$	0.0242	0.0034	0.9993	0.9826	0.9905
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.9877	0.9983	0.0244	0.1343	0.0965
			$\kappa = 1.676$		
$\text{diag}(P^*_{t\|T})$	0.3259	0.0106	0.9963	0.6915	0.9756
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.8189	0.9945	0.0596	0.5595	0.1549

As in the previous versions of the LW model, the two shocks that make up the natural rate cannot be recovered. In the $\kappa = 9.033$ high parameter setting, the correlation between the true and estimated shocks is less than 10%, in fact, as low as 2.5% for the shocks of '*other*

[13] They are

$$\begin{aligned}\Delta E_T\eta_{5t} &= 0.1654\Delta E_T\eta_{3t} - 0.0028E_T\eta_{4t},\\ E_T\Delta r_t^* &= 4E_T\eta_{5t} + E_T\eta_{3t},\end{aligned}$$

and

$$\begin{aligned}E_T\Delta r_t^* = {} & E_T\Delta r_{t-1}^* - 0.0381E_T\eta_{1t} - 0.0003E_T\eta_{2t} - 0.0044E_T\eta_{4t}\\ & + 0.0180E_T\eta_{1t-1} - 0.0068E_T\eta_{4t-1}.\end{aligned}$$

factor' z_t. Recovery appears to be concentrated on the output gap and inflation equation shocks, while the shock to y_t^* is not recoverable. In the low $\kappa = 1.676$ parameter setting, the degree of recovery of the trend shock y_t^* increases at the expense of the output gap shock, which decreases. The natural rate shocks remain unrecoverable. The correlation between true and estimated Δr_t^* are, respectively, 0.0819 and 0.1396.[14]

In summary, in all three variants of the Laubach and Williams model the natural rate r_t^* cannot be recovered from the data. This message can be easily communicated to policymakers using the correlation coefficient between the true model implied value and the Kalman smoother estimate from the data. This is never above 0.1764, and can be as low as 0.0819.

3.2 Recovering the Neutral Real Rate and the NAIRU - McCririck and Rees (2017)

McCririck and Rees' model (2017, MR) is effectively an extension of LW and adds an equation for Okun's law to enable the determination of a number of macroeconomic stars. The stars of interest are: growth in potential GDP, the NAIRU, and the neutral real interest rate, denoted by g_t, u_t^* and r_t^*, respectively. The model takes the form:[15]

$$\tilde{y}_t = \alpha_1 \tilde{y}_{t-1} + \alpha_2 \tilde{y}_{t-2} - \frac{a_r}{2} \sum_{i=1}^{2} (r_{t-i} - r_{t-i}^*) + \sigma_1 \varepsilon_{1t} \tag{21}$$

$$\pi_t = (1-\beta_1)\pi_t^e + \frac{\beta_1}{3} \sum_{i=1}^{3} \pi_{t-i} + \beta_2 (u_{t-1} - u_{t-1}^*) + \sigma_2 \varepsilon_{2t} \tag{22}$$

$$\Delta z_t = \sigma_3 \varepsilon_{3t}, \tag{23}$$

$$\Delta y_t^* = g_t + \sigma_4 \varepsilon_{4t} \tag{24}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{25}$$

[14]Identities of the form in (18), (19), and (20) exist again also for the post COVID-19 HLW (2023) model.

[15]Note that in MR, g_t rather than g_{t-1} is in the potential GDP growth equation, and the sign of the interest rate variables in the IS equation has changed. Also, for ease of comparability, we use the shock numbering $\{\sigma_i \varepsilon_{it}\}_{i=1}^{7}$ as in LW, rather than the labelling used in MR.

$$\Delta u_t^* = \sigma_6 \varepsilon_{6t} \tag{26}$$

$$u_t = u_t^* + \beta(.4\tilde{y}_t + .3\tilde{y}_{t-1} + .2\tilde{y}_{t-2} + .1\tilde{y}_{t-3}) + \sigma_7 \varepsilon_{7t} \tag{27}$$

$$r_t^* = 4g_t + z_t, \tag{28}$$

where u_t is the unemployment rate, u_t^* the NAIRU, π_t^e is measured expected inflation, and the remaining variables are as before in LW.

In MR's model, there are three observables — output growth, inflation and the unemployment rate — and seven shocks. Thus, the full set of seven shocks will not be recoverable. Writing their model in a SSF as before, and using the posterior means reported in Table A2 of their paper, we obtain the following recovery measures shown in Table 4 below:

Table 4: Recovery measures: McCririck and Rees (2017) shocks

Shocks:	$\varepsilon_{1t}(\tilde{y}_t)$	$\varepsilon_{2t}(\pi_t)$	$\varepsilon_{3t}(z_t)$	$\varepsilon_{4t}(y_t^*)$	$\varepsilon_{5t}(g_t)$	$\varepsilon_{6t}(u_t^*)$	$\varepsilon_{7t}(u_t)$
$\text{diag}(P_{t\|T}^*)$	0.4621	0.0309	0.9740	0.1690	0.9528	0.9691	0.4421
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.7335	0.9846	0.1637	0.9126	0.2148	0.1772	0.7500

So, while there are issues in recovering the NAIRU shock ε_{6t}, the biggest concern is still the recovery of the neutral rate, since the entries corresponding to the two shocks that define r_t^* (ε_{3t} and ε_{5t}) still strongly indicate a lack of recoverability. A dynamic correlation between the smoothed estimates of ε_{5t} and several of the other shocks is also apparent. Therefore, giving these shocks macroeconomic names or labels, and understanding what is driving the estimates of r_t^*, is difficult. The correlation between true and estimated change in the natural rate remains low at 0.1907.

3.3 Endogenous interest rates: can this change the outcome?

The ability of LW's model and its variants to recover the neutral real rate appears to be extremely limited. Prompted by this, it is natural to consider if the model could be modified to address this. One possibility is to introduce more observed variables into the system. One issue is that the catch-all nature of one of the shocks driving the neutral real rate, z_t, makes this rather non trivial. Another possibility is to modify the structure of the model. In the LW model, the policy rate was assumed to be exogenous, i.e., there was no equation, such as a standard Taylor rule, to explain its evolution. As Pagan and Wickens (2022) observed, this means that the LW model has some undesirable features.

To see some of these undesirable features, it is useful to consider the time-series properties of the series implied by LW's model. By definition, r_t^* is an integrated process of order one, $I(1)$ henceforth, since both g_t and z_t are $I(1)$ processes. Because there is no equation for r_t in LW, there is no mechanism in place to ensure that r_t^* and r_t co-integrate. If they do not co-integrate, then both the output gap, $\tilde{y}_t$, and inflation π_t will be $I(1)$. Since the goal of many central banks is to stabilize inflation, it is difficult to see how this can be achieved in a model where inflation is allowed to follow an $I(1)$ process, and there is no control rule to make it $I(0)$. A simple way to avoid this issue is to add a monetary rule to the LW model.

We can examine the effect of adding a policy rule within the MR model. As the latter was utilized in the MARTIN policy model of the Reserve Bank Australia (see Ballantyne *et al.*, 2020), it is natural to adopt their nominal interest rate rule, which implies for the real rate:

$$r_t = .7\,(r_{t-1} - \Delta\pi_t) + .3\,[r_t^* + (\pi_t - \bar{\pi}) - 2(u_t - u_t^*)] - \Delta_2 u_t + 1.19\varepsilon_{8t}, \tag{29}$$

where $\bar{\pi}$ denotes the inflation target. It is important to note here that adding (29) to the baseline MR model adds both an additional observed variable and also an additional (monetary policy) shock, ε_{8t}.

With this extended model, there are now four observed variables and eight shocks, implying once again a short system, and so not all shocks will be recoverable. Computing recovery diagnostics for the extended MR model with a policy reaction function yields the results shown in Table 5 below.

Table 5: Recovery diagnostics: McCririck and Rees (2017) with MP rule shocks

Shocks:	$\varepsilon_{1t}(\tilde{y}_t)$	$\varepsilon_{2t}(\pi_t)$	$\varepsilon_{3t}(z_t)$	$\varepsilon_{4t}(y_t^*)$	$\varepsilon_{5t}(g_t)$	$\varepsilon_{6t}(u_t^*)$	$\varepsilon_{7t}(u_t)$	$\varepsilon_{8t}(r_t)$
$\text{diag}(P^*_{t\|T})$	0.4521	0.0243	0.9550	0.1682	0.9487	0.9547	0.4421	0.0549
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.7423	0.9876	0.2112	0.9109	0.2266	0.2167	0.7456	0.9727

Focusing on the neutral real rate relevant entries under headings $\varepsilon_{3t}(z_t)$ and $\varepsilon_{5t}(g_t)$ in Table 5, the $\text{diag}(P^*_{t|T})$ values of 0.9550 and 0.9487 with corresponding population correlations of 0.2112, and 0.2266, respectively, are again evidence of a lack of recoverability of the neutral rate. In fact, these are little changed from those in Table 4. Thus, adding a policy rule to LW type models does not alter our finding of no recoverability.

Of course, this is only one possible modification to LW style models. Others may be more successful. Knowledge of the limited ability of these models to recover the neutral real rate should prompt researchers to consider alternatives.

4 Star Wars: Is there a Better Way to Recover Stars?

Even though one cannot recover stars from the models described above, perhaps one can get closer by using a different structural representation or filter. In the context of our metaphor of stars being used as a guide in a journey, it might be possible to think of this strategy as devising a better star map in order to get a more precise view of the location of the stars. To investigate this, we consider in the next sub-section a model recently proposed for estimating neutral real rates by Schmitt-Grohé and Uribe (2022).

Rather than use a different structural model or map, one might get a clearer view of the stars with a different telescope. In particular one might define the star variable via a Beveridge-Nelson (1981, BN) decomposition, and we consider two applications of this approach. The first was recently advocated by Morley, Tran and Wong (MTW, 2023), and a second by Lubik and Matthes (2015, LM). The latter employ a finite-horizon version of the BN decomposition linked to a TVP VAR model for variables related to the star.

4.1 New Structural Models - Schmitt-Grohé and Uribe (2022) and (2024)

Schmitt-Grohe and Uribé (2022, SGU) present a new structural model for estimating the neutral real rate; a closely related model is used in Schmitt-Grohé and Uribe (2024) for finding trend inflation. It assumes that the log level of per capita output y_t is driven by two permanent stochastic components, x_t and x_t^r, which represent technology and non-monetary factors affecting the real interest rate. Inflation π_t is $I(1)$ and its permanent component is the nominal inflation target. Lastly, the nominal interest rate is $I(1)$, and it is driven by two permanent components — the inflation target and the non-monetary real rate permanent component. This model is related to Uribe's (2022) model on the Neo-Fisher effect in which inflation and the nominal interest rate co-integrate with a permanent monetary shock (i.e., the inflation target). Denoting the transitory (gap) components in these variables with a tilde, these can be expressed as:

$$\begin{aligned}
\tilde{y}_t &= y_t - x_t - \delta x_t^r \\
\tilde{\pi}_t &= \pi_t - x_t^m \\
\tilde{i}_t &= i_t - (1+\alpha) x_t^m - x_t^r.
\end{aligned}$$

The neutral real rate is taken to be a combination of the permanent components driving inflation, x_t^m, and x_t^r, although in their final model they set $\alpha = 0$ producing $r_t^* = x_t^r$. This is different to LW's natural rate specification in equation (8f), which takes the form $r_t^* = c4g_t + z_t$. In LW, r_t^* responds to growth in potential GDP (g_t) coming from technology, as well as to one "other" real non-monetary shock (z_t). In contrast, the SGU specification has no role for technology shocks to affect r_t^*.

To analyze the properties of SGU's model, we define $\Phi_t = \left[\begin{array}{ccc} \tilde{y}_t & \tilde{\pi}_t & \tilde{\imath}_t \end{array} \right]'$ and $\xi_t = \left[\begin{array}{ccccc} \Delta x_t^m & \tau_t^m & \Delta x_t & \tau_t & \Delta x_t^r \end{array} \right]'$, where τ_t^m and τ_t are stationary monetary and real shocks. Then the dynamics for the gaps Φ_t are described by the following Vector AutoRegression (VAR) equation:

$$\Phi_t = B\Phi_{t-1} + C\xi_t,$$

while the observation equations are:

$$\Delta y_t = \Delta\tilde{y}_t + \Delta x_t + \delta\Delta x_t^r + \sigma_y \varepsilon_t^y \tag{30}$$

$$\Delta\pi_t = \Delta\tilde{\pi}_t + \Delta x_t^m + \sigma_\pi \varepsilon_t^\pi \tag{31}$$

$$\Delta i_t = \Delta\tilde{\imath}_t + (1+\alpha)\Delta x_t^m + \Delta x_t^r + \sigma_i \varepsilon_t^i, \tag{32}$$

where ε_t^y, ε_t^π and ε_t^i are measurement errors. As the observed variables are first differences, these measurement error shocks will have a permanent impact. Notice that even without the measurement errors the system is short, having three observables and five shocks.

The shocks are assumed to evolve as AR(1) processes. Of importance here is:

$$\Delta x_t^r = \rho_5 \Delta x_{t-1}^r + \sigma_5 \varepsilon_{5t}. \tag{33}$$

SGU estimate the system parameters by Bayesian methods. Some of the entries in C are fixed at values needed for identification of the parameters.[16]

It should be clear that, to recover r_t^*, one needs to be able to recover ε_{5t} in (33). Including the measurement errors, there are eight shocks and three observed variables, which means not all of the shocks can be recovered. From Table 6 it appears that only the neutral real rate shock ε_{5t} might be recovered, as the value of around .16 could be viewed as close to zero.

[16]We thank Martín Uribe for providing the posterior mean parameter estimates (see the GitHub repository). We put $\alpha = 0$ as we did not receive a posterior mean for it; the posterior median reported is very close to 0.

Table 6: Recovery measures: Schmitt-Grohé and Uribe (2022) shocks

Shocks:	ε_{1t}	ε_{2t}	ε_{3t}	ε_{4t}	ε_{5t}	ε_t^y	ε_t^π	ε_t^i
$\text{diag}(P^*_{t\|T})$	0.3949	0.7873	0.6211	0.5441	0.1596	0.9163	0.7457	0.8310
$\text{Corr}(\varepsilon_t, E_T\varepsilon_t)$	0.7779	0.4612	0.6155	0.6752	0.9168	0.2893	0.5043	0.4111

One might ask here how recovery of the real rate shock changes with the sensitivity of the output gap to the real rate shock. This is captured by the parameter δ in (30) as its value determines how important technology shocks are relative to "other" real shocks.[17] At the extreme, when $\delta = 0$, a much higher value for $P^*_{t|T}$ of .83 is found for ε_{5t}, indicating that the shock cannot be recovered. This points to a fundamental role for δ in the recovery of this shock in this model. To examine this more closely, take the equation for the output gap:

$$\tilde{y}_t = b_{11}\tilde{y}_{t-1} + b_{12}\tilde{\pi}_{t-1} + b_{13}\tilde{\imath}_{t-1} + c_{11}\Delta x_t^m$$
$$+ c_{13}\Delta x_t + c_{14}z_t + c_{15}\Delta x_t^r.$$

There is a measurement equation involving observed output growth Δy_t that is given by:

$$\Delta y_t = (b_{11} - 1)\tilde{y}_{t-1} + b_{12}\tilde{\pi}_{t-1} + b_{13}\tilde{\imath}_{t-1} + c_{11}\Delta x_t^m$$
$$+ c_{13}\Delta x_t + c_{14}z_t + (c_{15} + \delta)\Delta x_t^r + \sigma_y\varepsilon_t^y$$
$$= \eta_t + (c_{15} + \delta)\Delta x_t^r.$$

Suppose now that $\rho_5 = 0$ in equation (33). Then, η_t is uncorrelated with Δx_t^r. Moreover, δ does not affect the variance of this latter variable. Hence the variance of Δy_t would vary directly with δ, once all other parameters are set (e.g. to the posterior mean). This gives rise to two interesting observations. First, the posterior mean of c_{15} is very small ($-.0051$). If it was zero, then the model variance of Δy_t will depend on δ^2. This may explain why

[17] In SGU's paper, the posterior median of δ is 8.6, which is very similar to its mean of 8.3292 which we use.

SGU found that there was some evidence of counter-intuitive *negative* values for δ. Indeed, setting $\delta = 8.3292$ (the posterior mean) produces standard deviations of Δy_t, Δp_t and Δi_t of 4.67, 1.63 and 1.32, whereas putting $\delta = -8.3292$ we similarly get 4.65, 1.63 and 1.32.

Secondly, the fraction of the variance of Δy_t explained by the real rate shock ε_{5t} will rise as δ rises. Thus, when $\delta = 8.3292$ we find that nearly 80% of the variation in GDP growth is due to neutral real rate shocks. This appears to be rather high, since these are shocks that, as Schmitt-Grohé and Uribe (2022, p. 4) write: "*could stem from, for example, secular variations in demographic variables, exogenous changes in subjective discount rates, or in other factors determining the domestic or external willingness to save*". To reduce this influence it is necessary to reduce the magnitude of δ. Indeed, if $\delta = 2$, holding all other parameters unchanged, the real neutral rate shocks explain 18% of output growth and, with that value, the diag$(P^*_{t|T})$ entry for the fifth shock ε_{5t} is .73, indicating that it cannot be recovered. Clearly, the issue here is whether we have strong opinions about the likelihood of these "other" real shocks driving so much of growth, while technology determines so little, as $\delta = 8.3292$ implies.

Why does one get such a high δ estimate from the model? Fundamentally, δ is a free parameter that enables the model to better match the output growth data. To see this, note that the standard deviation of the GDP growth data is 4.89. Setting $\delta = 8.3292$ leads to a model based value of the standard deviation of GDP of 4.67, which matches the data well. If instead, $\delta = 2$, there is a standard deviation of GDP growth of 2.37 — a poor match. As δ rises, a larger proportion of output growth is accounted for by the real neutral rate shock, making recovery of that shock from the data easier.

4.2 A Different Telescope - The Beveridge-Nelson Filter

The Beveridge-Nelson (BN) decomposition has been used in several ways to estimate stars. Morley *et al.* (2023, MTW) is a recent approach. They define the star variable as the permanent component of a series found with the BN decomposition. This is a sensible proposal, but there are possible short system issues which we investigate in the first sub-section that follows below. An earlier proposal using BN was Lubik and Matthes (2015, LM) who estimate a simple TVP-VAR for three variables to find the neutral real rate. They deviate from the standard BN decomposition by working with a time horizon of five years, rather than an infinite one, when defining the permanent component as the *'long-run'* forecast. Again, there are short system issues that we discuss.

4.2.1 The MTW (2023) BN Approach

MTW's strategy consists of three steps to estimate the star variable of interest, which is the real neutral rate r_t^*. Unlike other studies, MTW treat the real rate of interest r_t as latent and define the observable real rate as $\tilde{r}_t = r_t + vm_{1t}$, where vm_{1t} is an $I(0)$ measurement error, uncorrelated with r_t.[18] There are other observable variables in the system. To briefly summarize the MTW approach, we use the data generating process of their simulation example in Section 3.3, which contains one additional observable variable $\tilde{x}_t$ that is similarly related to x_t via measurement error vm_{2t}.

First, an assumption is made about the behaviour of the latent variables r_t and x_t. In their simulations, these variables follow a VAR(1) of the form:

$$\underbrace{\begin{bmatrix} \Delta r_t \\ \Delta x_t \end{bmatrix}}_{\Delta z_t} = \underbrace{\begin{bmatrix} 0 & -.05 \\ 0 & .95 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta r_{t-1} \\ \Delta x_{t-1} \end{bmatrix}}_{\Delta z_{t-1}} + \underbrace{\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}}_{v_t} \tag{34}$$

[18]It is unclear why the measurement error is on the level of r_t, rather than on the growth rate Δr_t, since it would become less and less important as the sample size grows. Nonetheless, the same analysis that we provide below would still apply if it was on Δr_t.

where

$$\underbrace{\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}}_{v_t} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} .1125 & .1 \\ .1 & .1 \end{bmatrix}}_{V}\right). \tag{35}$$

The BN definition of the permanent components corresponding to (34), denoted with a superscript p, is given by:

$$\Delta z_t^p = (I - A)^{-1} v_t, \tag{36}$$

yielding the individual equations:

$$\Delta r_t^p = v_{1t} - v_{2t} \tag{37a}$$

$$\Delta x_t^p = 20 v_{2t}. \tag{37b}$$

The relations in (37) are the permanent components of the (multivariate) BN decomposition of Δz_t, where Δr_t^p in (37a) is the BN estimate of Δr_t^*.

In their second step, because $\tilde{r}_t$ and $\tilde{x}_t$ are observables and r_t and x_t are not, all the variables are connected by measurement errors specified as $vm_t = \sqrt{0.05}\, v_t$, where v_t is defined in (35). This leads to the system:

$$\Delta \tilde{r}_t = \Delta r_t + \Delta vm_{1t}$$

$$\Delta \tilde{x}_t = \Delta x_t + \Delta vm_{2t},$$

which implies

$$\begin{aligned} \Delta \tilde{z}_t &= \Delta z_t + \Delta vm_t \\ &= (I - AL)^{-1} v_t + \Delta vm_t. \end{aligned}$$

Consequently, the BN estimate of the permanent component in terms of observables $\Delta\tilde{z}_t$ is:

$$\Delta\tilde{z}_t^p = (I - A)^{-1} v_t. \tag{38}$$

Comparing (38) to (36), one can see that the shocks driving the permanent components of $\tilde{z}_t$ and z_t are the same.

MTW assume that the researcher mistakenly lets $\Delta\tilde{r}_t$ and $\Delta\tilde{x}_t$ follow a VAR(1) process, as was true of Δr_t and Δx_t, when getting a preliminary BN estimate of the permanent component r_t^*.[19] This '*preliminary BN*' estimate $\Delta\tilde{r}_t^*$ is given by:

$$\Delta\tilde{r}_t^* = 1.06\tilde{v}_{1t} - .949\tilde{v}_{2t}. \tag{39}$$

Note that there is serial correlation in $\Delta\tilde{r}_t^*$ (its first order auto-correlation coefficient is $-.15$).

Finally, since $\Delta r_t^* = v_{1t} - v_{2t}$ (the permanent component in (37a) from the VAR(1) specification) is a white noise process, and $\Delta\tilde{r}_t^*$ in (39) is not, MTW proceed to find an estimator of Δr_t^* in the third step which has that property. They describe this as '*robust to misspecification*', where the misspecification term refers to the presence of measurement error. To produce their '*robust*' estimator of Δr_t^* ($\Delta\hat{r}_t^*$), they assume an AutoRegressive Moving Average (ARMA) process for $\Delta\tilde{r}_t^*$, and then derive the new estimate $\Delta\hat{r}_t^*$ from the BN solution for that process. Fitting an ARMA(1, 2) model to $\Delta\tilde{r}_t^*$ gives:

$$\Delta\tilde{r}_t^* = .377\Delta\tilde{r}_{t-1}^* + \omega_t - .620\omega_{t-1} + .071\omega_{t-2},$$

where ω_t is white noise. MTW then define the robust estimate of the BN permanent component as $\Delta\hat{r}_t^* = \frac{1-.620+.071}{1-.377}\hat{\omega}_t = .72\hat{\omega}_t$, and its standard deviation is $.72 \times .152 = .11$. By

[19]The VAR(1) coefficient estimates are inconsistent since $\Delta\tilde{z}_t$ is a Vector Autoregressive Moving Average (VARMA) process, and not a VAR. To find the large sample estimates of the VAR(1) coefficients, we simulate 50,000 observations from their VARMA model and fit a VAR(1) to the simulated data.

construction, this approach produces an estimate with the property that $\Delta\hat{r}_t^*$ is white noise. However, $\Delta\hat{r}_t^*$ is *not* Δr_t^*. The correct BN permanent shock is $v_{1t} - v_{2t}$, which has a standard deviation of .11. Regressing this against $\hat{\omega}_t$ gives a recovery measure R^2 of .61. This illustrates that, while MTW is more successful than Laubach and Williams (2003) and its related approaches, one cannot fully recover the actual permanent shock with this strategy.

For comparison, a regression of the correct BN permanent shock against the *preliminary value* $\Delta\tilde{r}_t^*$ yields an R^2 of .58, and this preliminary value is more volatile (its standard deviation is .157). This highlights that their correction improves the estimate of the variance of the correct BN permanent shock. However, its robustness is limited to producing an estimate for the change in the real neutral rate which is white noise and it does not fully recover Δr_t^*.

To understand why this is the case it is useful to determine what drives $\hat{\omega}_t$. Regressing $\hat{\omega}_t$ against current and ten lags of v_{1t}, v_{2t}, vm_{1t} and vm_{2t} gives an $R^2 = .9998$, i.e., this virtually is an identity. When the terms vm_{1t} and vm_{2t} are excluded, the R^2 drops to .7, indicating that the measurement errors are very informative in the computation of $\hat{\omega}_t$, and therefore the robust estimate, in contrast to the true BN decomposition. The importance of measurement errors to the robust estimate Δr_t^* contributes to its recovery R^2 being .61.

4.2.2 Recovering Stars using Time-Varying Parameter Models

Another way the BN decomposition has been used to estimate stars is to couple it with a Time-Varying Parameter (TVP) model. As an example, consider the study by Lubik and Matthes (2015, LM) who estimate a simple TVP-VAR for three variables: the growth rate of real GDP, the PCE inflation rate, and the same real interest rate as in LW (2003). This measure is regularly updated and published by the Federal Reserve Bank of Richmond.[20] Their proposal is to measure the natural real rate of interest as the (conditional) long-horizon forecast of the observed real rate, so it is a variant of the BN definition of the permanent

[20]See https://www.richmondfed.org/research/national_economy/natural_rate_interest.

component. In their paper, the chosen time horizon is five years.

To illustrate the issues with such an approach, consider a simpler TVP model for a single equation only, the real interest rate, consisting of:

$$r_t = \rho_t r_{t-1} + \sigma_1 \varepsilon_{1t} \tag{40}$$

$$\Delta\rho_t = \sigma_2 \varepsilon_{2t}, \tag{41}$$

where ε_{1t} and ε_{2t} are mutually and serially uncorrelated, with zero mean and unit variance. Suppose, for simplicity that we define r_t^* as the prediction of r_t two periods ahead (instead of the five used in LM), that is, $r_t^* = E_t r_{t+2}$. Then, using (40) and (41):

$$\begin{aligned} r_t^* &= E_t(\rho_{t+2} r_{t+1} + \sigma_1 \varepsilon_{1t+2}) \\ &= E_t[(\rho_t + \sigma_2\varepsilon_{2t+2} + \sigma_2\varepsilon_{2t+1}) r_{t+1} + \sigma_1\varepsilon_{1t+2}] \\ &= E_t[(\rho_t + \sigma_2\varepsilon_{2t+2} + \sigma_2\varepsilon_{2t+1})(\rho_{t+1} r_t + \sigma_1\varepsilon_{1t+1}) \\ &= E_t[(\rho_t + \sigma_2\varepsilon_{2t+2} + \sigma_2\varepsilon_{2t+1})(\rho_t + \sigma_2\varepsilon_{2t+1}) r_t)] \\ &= E_t(\rho_t^2 + \sigma_2^2) r_t. \end{aligned} \tag{42}$$

Now, in the above, all random variables observed at time t are known, but future ones are unknown and are replaced by their unconditional means of zero, i.e., $E_t(\varepsilon_{1t+i}) = 0, \forall i > 0$. It then needs to be recognized that, while r_t is known, ρ_t is not, and the expectation must be conditional on the data. The relation in (42) then leads to a star type of estimate of the real rate r_t^* having the form:

$$r_t^* = r_t E_t(\rho_t^2) + \sigma_2^2 r_t. \tag{43}$$

The problem then is that $E_t(\rho_t^2)$ is not computed by the Kalman filter. To proceed, Lubik and Matthes (2015) did something different. For this case their approach would be to measure

r_t^* as $r_t E_t(\rho_{t+2})$, and *not* $r_t E_t(\rho_t^2) + \sigma_2^2 r_t$ in (43), as implied by the TVP model.

More generally, typically in a TVP VAR there will be shocks that would drive the structural equations and shocks that determine the evolution of the TVPs. So, as discussed in Pagan and Robinson (2022), the system is short. Consequently, there will be linear relations between at least some of the filtered quantities. The extent of which this impedes the recovery of the star variable will vary across TVP models, pointing to a need for it to be reported.

5 Stochastic Volatility Can Obscure the Stars

We now turn to a recent feature of many modern macroeconomic models whose potential to impede shock recovery, and therefore obscure the stars, does not appear to be appreciated; namely, Stochastic Volatility (SV).[21] Initially SV was included in models used to summarize the data; a prominent macroeconomic example is the univariate model of U.S. inflation by Stock and Watson (2007). More recently, SV has increasingly been included in models which interpret the economy through shock estimates and impulse responses; for example, the SVAR of Mumtaz and Zanetti (2013) is a prominent case. An example of the inclusion of SV in models intended to estimate stars is Beyer and Milivojevic (2023), who estimate the neutral real rate for 50 countries.

To see how recovery issues may materialize, consider the following simple example. Suppose that there is a single variable and it has conditional volatility that is specified to follow an SV process. This produces the following model:

$$y_t = B_1 y_{t-1} + \exp\{.5h_t\}\varepsilon_t \tag{44a}$$

$$h_t = \mu + \beta h_{t-1} + \omega_t. \tag{44b}$$

[21] A second recent feature often included are news shocks, following Beaudry and Portier (2004 and 2006). The consequences of news shocks for shock recovery are discussed in the working paper version of this paper. Suffice it to say that the systems featuring them are short.

Although estimation of the parameters can be complex and is important in practice, as before we assume we have parameter estimates or know their true values. Then, define:

$$\zeta_t = y_t - B_1 y_{t-1}$$
$$\Leftrightarrow \zeta_t^2 = \exp\{h_t\}\varepsilon_t^2,$$

so that

$$\log(\zeta_t^2) = h_t + \log(\varepsilon_t^2)$$
$$= \mu + \beta h_{t-1} + \omega_t + \log(\varepsilon_t^2). \tag{45}$$

Computing smoothed shocks gives an SSF with equations:

$$\log(\zeta_t^2)^D = \mu + \beta E_T h_{t-1} + E_T \omega_t + E_T \log(\varepsilon_t^2) \tag{46}$$
$$E_T h_t = \mu + \beta E_T h_{t-1} + E_T \omega_t. \tag{47}$$

Because there is only one observable $\log(\zeta_t^2)$ in (44), the system is short and both the shocks ε_t and ω_t cannot be recovered.[22] Note here again that it has been assumed that parameters are either known or estimates of them are available. The SSF *has* to hold — it is an implication of the SV model. The inclusion of SV therefore can be problematic when the model is used to interpret, rather than summarize, the data with the shocks, as occurs when estimating stars.

Is there an alternative to the SV specification? Yes, of course. Other major classes of models for capturing conditional volatility, namely (E)GARCH (Bollerslev, 1986, and Nelson, 1991), are not short and are well capable of capturing the same type of time varying volatility behaviour in macroeconomic variables as the SV model. For example, one might

[22]This scenario is exactly the same as the one in Section 2.2, albeit with the second shock being log transformed.

use an EGARCH model taking the form:

$$y_t = B_1 y_{t-1} + \exp\{.5h_t\}\varepsilon_t$$
$$h_t = \mu + \beta h_{t-1} + \alpha\varepsilon_{t-1}^2,$$

and avoid the recoverability issues introduced by a SV process.[23]

6 Estimating Stars Without Short Systems: Smooth-Transition Models

A common feature of many models intended to estimate stars is that the stars are modelled as evolving via an exogenous non-stationary process, or as a function of several such processes. This approach is often motivated as being agnostic, allowing shifts in the star to occur without a stance being taken on either when they took place or how the star variable changed between these shifts. Instead, it is constantly changing over time. However, as shown above, this flexibility can limit the ability to recover the stars and hence the interpretation of what drives them.

Okimoto (2019), is an example of an alternative approach to modelling a star that does not result in a short system by using a smooth-transition model (see van Dijk *et al.*, 2002 for a survey). These allow for a finite number of changes in the star. Okimoto (2019), uses a smooth transition model to describe the evolution of the star variable trend inflation π_t^*. With a sample of T observations, the aim of this approach is to capture the evolution of the star as undergoing a smooth transition from the value at the beginning of the sample μ_1 to that at the end μ_2 using a deterministic function that depends on (t/T).[24] There are many

[23] Hamilton et al. (2016) estimate a long-run world real rate drawing on individual rolling ARCH(2) models of the real rate for a range of countries.

[24] One could allow for knot points in the sample as well, just as one does with spline functions.

such functions that could be applied, one of which is the exponential function employed by Okimoto (2019):

$$\pi_t^* = \mu_1 + G(s_t; c, \gamma)(\mu_2 - \mu_1) \tag{48}$$
$$G(s_t; c, \gamma) = \frac{1}{1 + \exp(-\gamma(s_t - c))}, \gamma > 0$$
$$s_t \equiv \frac{t}{T}.$$

Smooth-transition models have been used to model stars other than trend inflation. For example, Murphy (2020) does so for the NAIRU in the context of a large macroeconomic model of Australia, albeit with a different transition function $G(\cdot)$ than in (48); Lye and McDonald (2021) also has elements of this. Recently Gao *et al.* (2024) have proposed a related approach for TVP structural VAR models.

We believe that further analysis of this approach to modelling star variables such as through the evaluation of their real-time reliability — akin to Orphanides and Van Norden (2002) for unobserved-component models — is warranted in order to better understand their potential usefulness for policy.

7 Conclusion

Stars are frequently cited in speeches by central bank officials and the financial press when addressing the appropriateness of the current policy stance. Moreover, estimates of stars are routinely published by central banks and organizations such as the OECD in their Economic Outlook report. Recently, authors from the World Bank have produced an extensive cross-country database of stars such as the growth rate of potential output (see Kilic Celik *et al.*, 2023). In general, substantial resources are devoted to estimating stars, which highlights their importance in the conduct of macroeconomic policies.

Federal Reserve Chairman Jerome H. Powell once commented that conventional wisdom is that monetary policy involves navigating by stars like ships of the past, but shifting stars makes that challenging (Powell, 2018). In that regard, Sablik (2018, p. 3) records that New York Fed President John C. Williams (one of the authors of the LW model) bemoaned the challenges of using the natural real rate as a guide for policy by saying: *'As we have gotten closer to the range of estimates of neutral what appeared to be a bright point of light is really a fuzzy blur'*. These comments illustrate some of the issues arising that relate to parameter uncertainty, shifts in stars, and wide confidence intervals surrounding estimates of stars. They significantly complicate the conduct of macroeconomic policy. And they are well known.

The point of this article is more fundamental. Drawing on the recent theoretical literature on shock recovery, we simply ask whether the models used to estimate stars can in fact recover the true star from the observed data. This would seem to be a minimal desirable property of any model. We address this question in the most favorable setting conceivable, namely, when the models used to measure the star variables are correctly specified, all their parameters are known, and an infinite amount of the observed series are available. The answer to this question is that the ability to recover stars varies considerably across the models. In the workhorse Laubach and Williams model, for example, it is not possible to recover the main star variable of interest, r_t^*.

Understanding the limitations of models which play a critical role in the conduct of macroeconomic policy is important. Whether a model can recover the variable it is intended to measure is paramount, yet it is not routinely discussed. Just as presenting confidence or highest posterior density intervals around stars is standard practice for demonstrating the statistical uncertainty surrounding the estimates, the extent of recoverability of the star variable also needs to become standard disclosure information. We have shown how this can be communicated simply as a correlation between the estimated (first difference) in the star variable and its true population value, which is easily calculated using the Kalman filter and smoother. This correlation should be routinely reported alongside star estimates to policymakers. Knowledge of how recoverability varies across models will assist policy makers discern amongst them.

One conclusion from this paper is that our ability to navigate economic policy by the stars is even more limited than we thought. A second is that knowing the extent to which a model can recover its star variable — essentially, understanding its limitations — is useful for researchers. It can help them understand whether further model development is desirable. There are many possible directions for this development, such as considering alternative structures, including more observed information, or moving away from handling star variables as an exogenous stochastic variable. More generally, there is a trend to incorporate greater flexibility into macroeconomic models, frequently by introducing additional shocks, and these inevitably lead to short systems. While the aim of providing a better description of the data is admirable, it is necessary to recognize that this has limitations. The stars the model was intended to shine light on can be obscured.

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