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The term structure of interest rates in an estimated New Keynesian policy model[☆]

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ABSTRACT

We jointly estimate a New Keynesian policy model with a Gaussian affine no-arbitrage specification of the term structure of interest rates, and assess how important inflation, output and monetary policy shocks are as sources of fluctuations in interest rates and the term premium. We work with observable pricing factors and utilize the computationally convenient normalization of Joslin et al. (2013b). This allows us to estimate the model without needing to restrict the parameters driving the market prices of risk. Using data for the U.S. from 1962:Q1 to 2014:Q2, we find that inflation and the output gap account for around 80% of the unconditional forecast error variance of bond yields at the short and medium end of the term structure, while monetary policy shocks account for around 20%. Bond yields respond to macroeconomic shocks only gradually, peaking after about 4 quarters. This is due to sizable monetary policy inertia estimates in our model. At the peak of the response, inflation shocks increase bond yields by more than one-to-one, and output shocks by less than one-to-one, which is consistent with a Taylor type monetary policy rule. Our term premium estimate is strongly counter-cyclical and can capture salient features of the term structure that constitute a puzzle in the expectations hypothesis.

1. Introduction

Since the seminal work by Ang and Piazzesi (2003), interest in so called *'macro-finance models'* which incorporate macroeconomic variables into an affine Gaussian no-arbitrage term structure model to assess the effects of macroeconomic information on the yield curve has increased tremendously.[1] Studies that analyze macro-finance relations using such a structural approach are, among many others, (Rudebusch and Wu, 2008; Hördahl et al., 2006; Bekaert et al., 2010) and Bikbov and Chernov (2013). These studies formulate small New Keynesian type models jointly with a dynamic no-arbitrage model for bond yields. Rudebusch and Wu (2008); Hördahl et al. (2006), Bekaert et al. (2010) summarize interest rate movements with inflation, output and two latent yield curve factors, while Bikbov and Chernov (2013) use inflation, output and the short rate. In all of these studies, some or all of the parameters that determine the prices of risk are restricted to zero. For instance,

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[1] There exist earlier studies that have modelled the relationship between yields and macroeconomic variables, nevertheless *without* imposing no-arbitrage restrictions on bond prices.

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Rudebusch and Wu (2008) restrict risk prices such that only entries related to the latent factors are non-zero. Hördahl et al. (2006) and Bikbov and Chernov (2013) estimate their model first without restrictions on the price of risk, and then re-estimate the model with all the *'insignificant'* parameters found in the first step set to zero. Bekaert et al. (2010) restrict *all* risk prices to zero, such that the Expectations Hypothesis holds exactly. While these zero-restrictions enable the authors to mitigate some of the well known estimation difficulties inherent in these models, such *'arbitrary'* zero-restrictions produce numerically and economically significant differences in policy relevant outcomes related to term premia, inflation expectations and the like. In this context, Bauer (2014a) has recently pointed out, that the approach by Hördahl et al. (2006) and Bikbov and Chernov (2013) might be unappealing for several reasons: First, choosing restrictions based on individual standard errors amounts to imposing a joint restriction without considering joint significance. Second, the choice of significance is necessarily arbitrary. Third, different choices of zero-restrictions on the risk factors lead to economically significant differences in policy relevant variables, with their two-step approach providing no guidance with respect to which results are deemed more credible. Finally, the approach of Bekaert et al. (2010) may seem difficult to justify empirically because there exists strong evidence that the Expectations Hypothesis is rejected in the data (see, for instance, the seminal papers by Campbell and Shiller (1991) and Cochrane and Piazzesi (2005)). Recent reduced-form studies that emphasize the importance of the choice of zero restrictions on the prices of risk are Joslin et al. (2014) and Bauer (2014a).

Apart from the problems related to the imposition of arbitrary zero restrictions on the prices of risk in previous macro-finance terms structure models, there exist also some concerns related to the empirical specification of the New Keynesian type macroeconomic models that are employed. For instance, both, Hördahl et al. (2006) and Rudebusch and Wu (2008), work with monthly data for Germany and the U.S., respectively. Due to difficulties in adequately capturing the empirically observed dynamics in the data when switching to a monthly sampling frequency, these authors modify the original lag specifications in the New Keynesian models and add an arbitrary number of lags to the inflation and output gap equations in their formulations.[2] Moreover, as GDP is only published on a quarterly basis, Rudebusch and Wu (2008) work with industrial capacity as a measure of the output gap. In this paper, we work with a quarterly time horizon. This enables us to use standard GDP based measures of the output gap. Furthermore, we bypass the need to modify the lag structure describing the dynamics in New Keynesian models, which allows us to relate the parameter estimates that we obtain to those found in, for instance, Cho and Moreno (2006) or in Buncic and Melecky (2008). These two studies estimate New Keynesian type policy models without an arbitrage free dynamic term structure model for bond yields, and thus can be used as a reference point for our structural macro parameter estimates.

The objective of this study is to estimate a Gaussian macro-finance term structure model using a small scale New Keynesian policy model to capture the dynamics of the macroeconomic variables and an affine no-arbitrage term structure model for bond yields. In order to mitigate computational difficulties inherent in the estimation of such models, we work with *observable* pricing factors only, and impose the computationally convenient normalizations suggested by Joslin et al. (2011) and Joslin et al. (2013b) to price the cross-section of bond yields. An advantage of our approach is that we do not need to impose any arbitrary restrictions on the parameters that determine the market prices of risk, yielding a more flexible parameterisation of our model. Our study thus contribute to the existing literature on structural Gaussian macro-finance term structure models (MTSMs) by estimating a model without imposing any zero restrictions on the prices of risk. Using this unrestricted framework, we provide an analysis of the effects of macroeconomic variables on bond yields and term premia.

Using quarterly data from 1962:Q1 to 2014:Q2 for the U.S., we find that inflation and the output gap account for about 80% of the total forecast error variance of yields at the short and medium ends of the yield curve, and that monetary policy shocks account for about 20%. Real economic activity explains (at most forecasting horizons) a larger part of the forecast error variance than inflation. Results for term premia lead to qualitatively similar results. Our impulse response function analysis shows that positive shocks to inflation and the output gap decrease the term premium instantaneously. In line with market participants' expectations, we find an instantaneous increase in the expectations hypothesis component of interest rates. These two effects offset each other initially, with the term premia responses reverting back to zero relatively quickly, while the responses of the expectations hypothesis term are longer lasting. The response of bond yields to macroeconomic shocks peaks after about 4 quarters, with inflation shocks increasing bond yields by more than one-to-one, while output shocks do so less than one-to-one.

Our term structure model implied pricing errors are on average around 65 basis points, with pricing errors increasing with maturity. These pricing errors are correlated with the slope of the yield curve, indicating that the inclusion of a factor correlated with the slope could reduce the pricing errors. Our estimated term premium is strongly counter cyclical, with a correlation of -0.83 between the term premium and real activity (the output gap). Moreover, our estimated term premium allows us to capture salient features of the term structure of interest rates that represent a puzzle for the Expectations Hypothesis, that is, LPY(i) and LPY(ii) of Dai and Singleton (2002), particularly for longer maturities.

The rest of the paper is organised as follows. Section 2 outlines in detail the structural macro-finance term structure model (MTSMs) that we estimate and the assumptions that we impose. In Section 3, we describe our estimation strategy and the data, with a discussion of the empirical results being presented in Section 4. Finally, we conclude our study in Section 5 and offer some extensions for future research.

[2] Note that most standard New Keynesian type dynamic macroeconomic models assume a quarterly time horizon so that the dynamics as well as parameters such as elasticities and discount rates are calibrated with a quarterly horizon in mind.

2. A structural MTSM

We follow the broad macro-finance literature and define the macroeconomic variables in our model to be spanned by the yield curve, so that our model implies a Taylor (1993) type policy rule (or response function) for the short rate. Before we proceed to explain the mechanics of our model, we briefly discuss what this assumption implies for our analysis.

There exists some empirical evidence to suggest that spanned macroeconomic variables have less explanatory power for future yield curve movements than unspanned macroeconomic variables. An important study providing support of this view is Ludvingson and Ng (2009). The authors show that when macroeconomic variables are included in the predictive regression of Cochrane and Piazzesi (2005), up to 44% of future bond returns can be explained by the model instead of around 35%, when only factors extracted from the yield curve are used. By working with an MTSM, which implies that macroeconomic variables are spanned by the yield curve, we potentially fail to capture this extra information in macroeconomic variables, as documented by Ludvingson and Ng (2009).

A possible interpretation of this unspanned part in macroeconomic variables is that a central bank is not willing to adjust its monetary policy stance at time t in response to all macroeconomic shocks that hit the economy at that point in time. Moreover, market participants should not expect the central bank to adjust its policy stance to all types of shocks. Although some shocks may not have a direct impact on current policy behavior of the central bank, they may still have an effect on future term premia, and therefore also on future excess bond returns. Such macroeconomic shocks are unspanned by the *current* yield curve, but indeed have an effect on future bond yields. Nevertheless, following the Federal Reserve's expressed view to adjust its policy rate at time t in response to movements in unemployment and inflation at time t, we assume that our macroeconomic variables are spanned by the yield curve.[3]

2.1. Modelling bond yields

Joslin et al. (2011) formulate a canonical Gaussian dynamic term structure model where pricing factors are observable linear combinations of yields.[4] The advantage of this modelling approach is that their normalization scheme is computationally convenient. In a related paper, Joslin et al. (2013b) extend the model of Joslin et al. (2011) to include spanned macroeconomic factors. In our study, we employ the normalizations of Joslin et al. (2013b) that allows for spanned macroeconomic factors, because of the computational convenience and tractability of this approach.

Before we outline the model, let us define the notation that will be used throughout the paper. Let $P_t^{(n)}$ denote the price of an n period zero coupon bond at time t, paying the holder 1$ at time $(t+n)$ when the bond matures, where the price is related to the (annualised) continuously compounded yield y_t as:

$$P_t^{(n)} = \exp\{-ny_t^{(n)}\}. \tag{1}$$

The (log) yield of the bond is defined as:

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}, \tag{2}$$

where $p_t^{(n)}$ denotes the log price, $\ln(P_t^{(n)})$. The (one period) log forward rate at time t for loans from time $(t+n-1)$ to $(t+n)$ is:

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}, \tag{3}$$

with the (holding period) return being:

$$hpr_t^{(n)} \equiv p_t^{(n-1)} - p_{t-1}^{(n)}. \tag{4}$$

The yield risk premium, that is, the average expected return from holding an n-period bond to maturity, financed by a sequence of one period bonds, is defined as:

$$yrp_t^{(n)} \equiv y_t^{(n)} - \frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t[y_{t+i}^{(1)}], \tag{5}$$

where $\mathbb{E}_t[\cdot]$ denotes the conditional expectation operator with information up to time t. The forward term premium, the expected return for planning today to borrow in the future spot market for a year, financed by contracting today to lend in the forward market is written as:

$$frp_t^{(n)} \equiv f_t^{(n)} - \mathbb{E}_t[y_{t+n}^{(1)}]. \tag{6}$$

Excess returns are defined as:

$$rx_t^{(n)} \equiv hpr_t^{(n)} - y_{t-1}^{(1)}, \tag{7}$$

where $y_t^{(1)}$ is the one period risk-free rate.

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[3] Note here that this literature entirely disregards the (potentially short lived) impact of central bank communication on the yield curve. For studies analysing the role of central bank communication, see Gürkaynak et al. (2005) and Brand et al. (2010).

[4] A canonical model is a maximally flexible model that puts a minimum number of restrictions on the loadings of the risk factors, such that the model is just identified.

2.2. Generic representation of a gaussian affine term structure model

To describe the dynamics of a generic Gaussian affine term structure model (ATSM), let $\mathbf{X}_t$ denote the $(k \times 1)$ dimensional state vector of risk factors that determines the cross-section of government bonds. The vector of risk factors is assumed to follow a VAR(1) processes of the form:

$$\mathbf{X}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{X}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_t, \tag{8}$$

where $\boldsymbol{\epsilon}_t$ is $(k \times 1)$ dimensional *i.i.d.* multivariate standard Normal random variable, and $\boldsymbol{\mu}$ and $\boldsymbol{\Phi}$ are, respectively, $(k \times 1)$ and $(k \times k)$ dimensional vectors and matrices that describe the evolution of $\mathbf{X}_t$ under the physical measure. The $(k \times k)$ dimensional matrix $\boldsymbol{\Omega} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}'$ captures the conditional variance/covariance of $\mathbf{X}_t$. The short rate is a function of the risk factors

$$r_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{X}_t, \tag{9}$$

where $r_t \equiv y_t^{(1)}$, that is, the one-period yield. The parameters δ_0 and $\boldsymbol{\delta}_1$ are of dimension (1×1) and $(k \times 1)$ and link the state variables $\mathbf{X}_t$ to the short rate r_t.

The nominal pricing kernel (or stochastic discount factor) that prices all nominal assets in the economy is denoted by $\mathcal{M}_{t+1}$. For a bond price to be arbitrage free, the gross return $R_{t+1}^{(n)} = P_{t+1}^{(n-1)}/P_t^{(n)}$ generated by holding the asset for one period must satisfy the following fundamental asset pricing relation:

$$1 = \mathbb{E}_t[\mathcal{M}_{t+1}(R_{t+1}^{(n)})] \tag{10}$$

$$\Leftrightarrow \; P_t^{(n)} = \mathbb{E}_t[\mathcal{M}_{t+1}(P_{t+1}^{(n-1)})]. \tag{11}$$

Thus, for a bond that matures in one period and pays 1$ at par, (11) becomes:

$$P_t^{(1)} = \mathbb{E}_t[\mathcal{M}_{t+1}], \tag{12}$$

while for all other maturities n we have the relation in (11).

In ATSMs, the pricing kernel is assumed to be conditionally log-Normal distributed, taking the form:

$$\mathcal{M}_{t+1} = \exp\left\{-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\epsilon}_{t+1}\right\}, \tag{13}$$

where $\boldsymbol{\lambda}_t$ is the $(k \times 1)$ dimensional vector of market prices of risk. Market prices of risk are also an affine function of the state vector $\mathbf{X}_t$:

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{X}_t, \tag{14}$$

where $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$ are of dimension $(k \times 1)$ and $(k \times k)$, respectively. Given this structure, a defining feature of ATSMs is that the state vector of factors $\mathbf{X}_t$ that determines the short rate r_t also determines all other bond prices (and yields) through the pricing kernel in the model. Moreover, following for instance see Appendix A (Ang and Piazzesi, 2003), it can be shown that this specification implies that the price of a zero coupon bond with maturity n is given by:

$$P_t^{(n)} = \exp\{A_n + \boldsymbol{B}_n'\mathbf{X}_t\}, \tag{15}$$

where the scalar A_n and $(k \times 1)$ vector $\boldsymbol{B}_n$ can be found recursively from the relations:

$$A_{n+1} = A_n + \boldsymbol{B}_n'(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0) + \frac{1}{2}\boldsymbol{B}_n'\boldsymbol{\Omega}\boldsymbol{B}_n - \delta_0 \tag{16}$$

$$\boldsymbol{B}_{n+1} = (\boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_1)'\boldsymbol{B}_n - \boldsymbol{\delta}_1, \tag{17}$$

for all $n = 2, 3, \ldots, J$, starting from $A_1 = -\delta_0$ and $\boldsymbol{B}_1 = -\boldsymbol{\delta}_1$. The yields corresponding to (15) are determined by:

$$y_t^{(n)} = a_n + \boldsymbol{b}_n'\mathbf{X}_t, \tag{18}$$

where $a_n = -A_n/n$ and $\boldsymbol{b}_n = -\boldsymbol{B}_n/n$.

Under the $\mathbb{Q}$ measure, the law of motion of the state vector $\mathbf{X}_t$ is also assumed to follow the same type of VAR(1) process as in (8), that is:

$$\mathbf{X}_t = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{X}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_t^{\mathbb{Q}}, \tag{19}$$

however, with a different set of parameters $\boldsymbol{\mu}^{\mathbb{Q}}$ and $\boldsymbol{\Phi}^{\mathbb{Q}}$ that govern its dynamics under $\mathbb{Q}$. The link from $\boldsymbol{\mu}^{\mathbb{Q}}$ and $\boldsymbol{\Phi}^{\mathbb{Q}}$ to the market prices of risk and physical parameters is given by the following two relations:

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0 \quad \text{and} \quad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_1. \tag{20}$$

The error term $\boldsymbol{\epsilon}_t^{\mathbb{Q}}$ in (19) is again assumed to be an *i.i.d.* multivariate standard Normal distributed random variable.

2.3. The Joslin et al. model

Joslin et al. (2013b) formulate an extension of the dynamic term structure model proposed in Joslin et al. (2011) (JSZ model henceforth), which allows for spanned macroeconomic factors to be included in the state vector. We start by briefly revisiting the JSZ model and then show how to incorporate macroeconomic factors in the model. The key to the JSZ model is the use of *observable* pricing factors that are linear combinations (or portfolios) of bond yields. Moreover, Joslin et al. (2011) show that the generic representation of the affine term structure model in Section 2.2 is observationally equivalent to a canonical model that takes the form:[5]

$$r_t = r_\infty^{\mathbb{Q}} + \mathbf{1}_k' \mathbf{Z}_t \tag{21}$$

$$\mathbf{Z}_t = \mathbf{\Psi}^{\mathbb{Q}} \mathbf{Z}_{t-1} + \mathbf{\Sigma}_z \boldsymbol{v}_t^{\mathbb{Q}}, \tag{22}$$

where the $(k \times 1)$ vector of state variables $\mathbf{Z}_t$ are *latent* (unobservable) and $\boldsymbol{v}_t^{\mathbb{Q}}$ is a $(k \times 1)$ dimensional vector *i.i.d.* multivariate standard normal random variable. The intercept term in (21) is denoted by $r_\infty^{\mathbb{Q}}$, and $\mathbf{1}_k$ is a $(k \times 1)$ dimensional vector of ones. The latent state vector $\mathbf{Z}_t$ is normalized to have a zero mean, so that no intercept is included in the relation in (22). Further, the $(k \times k)$ parameter matrix $\mathbf{\Psi}^{\mathbb{Q}}$ is also normalized in such a way that it contains its eigenvalues on the diagonal entries, that is, $\mathbf{\Psi}^{\mathbb{Q}} = \text{diag}(\boldsymbol{\lambda}^{\mathbb{Q}})$, where $\boldsymbol{\lambda}^{\mathbb{Q}}$ is the $(k \times 1)$ dimensional vector of eigenvalues of $\mathbf{\Psi}^{\mathbb{Q}}$, sorted from largest to smallest.

In the JSZ framework, yields of all maturities are a linear function of the *latent* state variables $\mathbf{Z}_t$, taking the form:

$$y_t^{(n)} = \tilde{a}_n + \tilde{\boldsymbol{b}}_n' \mathbf{Z}_t, \tag{23}$$

where $\tilde{a}_n = -\tilde{A}_n/n$ and $\tilde{\boldsymbol{b}}_n = -\tilde{\boldsymbol{B}}_n/n$, and

$$\tilde{A}_{n+1} = \tilde{A}_n + \frac{1}{2}\tilde{\boldsymbol{B}}_n' \Omega_z \tilde{\boldsymbol{B}}_n - r_\infty^{\mathbb{Q}} \tag{24a}$$

$$\tilde{\boldsymbol{B}}_{n+1} = \mathbf{\Psi}^{\mathbb{Q}\prime} \tilde{\boldsymbol{B}}_n - \mathbf{1}_k, \tag{24b}$$

are again derived recursively, starting from $\tilde{A}_1 = -r_\infty^{\mathbb{Q}}$ and $\tilde{\boldsymbol{B}}_1 = -\mathbf{1}_k$. Joslin et al. (2011) further assume that the variables of the observed state vector are linear combinations of yields defined as: $\mathbf{X}_t = \mathbf{W}\boldsymbol{y}_t$, where $\mathbf{W}$ is a $(k \times J)$ dimensional weight matrix, and $\boldsymbol{y}_t$ is the stacked vector of all J yields $y_t^{(n)}, \forall n = 1, 2, \ldots, J$, so that $\boldsymbol{y}_t = [y_t^{(1)}; y_t^{(2)}; \ldots; y_t^{(J)}]$ is of dimension $(J \times 1)$.[6]

In stacked vector form, the relation in (23) can be expressed as:

$$\begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \\ \vdots \\ y_t^{(J)} \end{bmatrix} = \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_J \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{b}}_1' \\ \tilde{\boldsymbol{b}}_2' \\ \vdots \\ \tilde{\boldsymbol{b}}_J' \end{bmatrix} \mathbf{Z}_t \tag{25}$$

$$\boldsymbol{y}_t = \tilde{\mathbf{a}}_{\mathbf{z}} + \tilde{\mathbf{b}}_{\mathbf{z}} \mathbf{Z}_t, \tag{26}$$

where $\tilde{\mathbf{a}}_{\mathbf{z}}$ and $\tilde{\mathbf{b}}_{\mathbf{z}}$ are of dimensions $(J \times 1)$ and $(J \times k)$, respectively. We can then express the observable states as a linear function of the latent states by the relations:

$$\mathbf{X}_t = \mathbf{W}\boldsymbol{y}_t$$

$$\mathbf{X}_t = \mathbf{W}(\tilde{\mathbf{a}}_{\mathbf{z}} + \tilde{\mathbf{b}}_{\mathbf{z}} \mathbf{Z}_t).$$

JSZ show that the affine term structure model described in Section 2.2 must satisfy the following relations when the state variables are defined to be linear combinations of yields:

$$\delta_0 = r_\infty^{\mathbb{Q}} - \boldsymbol{\delta}_1' \mathbf{W} \tilde{\mathbf{a}}_{\mathbf{z}}, \qquad \boldsymbol{\delta}_1' = \mathbf{1}_k' (\mathbf{W}\tilde{\mathbf{b}}_{\mathbf{z}})^{-1}, \tag{27a}$$

$$\boldsymbol{\mu}_{\mathbb{Q}} = (\boldsymbol{I}_k - \mathbf{\Phi}^{\mathbb{Q}}) \mathbf{W} \tilde{\mathbf{a}}_{\mathbf{z}}, \quad \mathbf{\Phi}^{\mathbb{Q}} = \mathbf{W}\tilde{\mathbf{b}}_{\mathbf{z}} \mathbf{\Psi}^{\mathbb{Q}} (\mathbf{W}\tilde{\mathbf{b}}_{\mathbf{z}})^{-1}, \tag{27b}$$

where $\tilde{\mathbf{a}}_{\mathbf{z}}$ and $\tilde{\mathbf{b}}_{\mathbf{z}}$ satisfy the recursions in (24) and $\boldsymbol{I}_k$ is the $(k \times k)$ dimensional identity matrix.

When macroeconomic variables are to be included in the pricing factors, Joslin et al. (2013b) propose an extension of the JSZ framework that can be used to estimate the model with macroeconomic information. Since the macroeconomic variables

[5] We use the same notation as in Joslin et al. (2011) to facilitate the exposition and comparison.

[6] Note here that we use the semicolon ‘;’ to denote the *stacking* operator (as in Matlab), so that $[a; b] = \begin{bmatrix} a \\ b \end{bmatrix}$.

are spanned by the yield curve, it follows that the macroeconomic variables are linear combinations (or portfolios) of yields. Let $\boldsymbol{M}_t$ denote the $(m \times 1)$ dimensional vector of macroeconomic variables. The portfolio weights can then be derived by defining $\mathbf{X}_t = [\boldsymbol{M}_t; \mathcal{P}_t^{\mathcal{L}}]$, and realizing that we can write $\mathbf{X}_t = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 \mathcal{P}_t^{\mathcal{N}}$, where $\mathcal{P}_t^{\mathcal{N}} = \mathbf{W}^{\mathcal{N}} \boldsymbol{y}_t$ and the parameters $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are defined by:

$$\underbrace{\begin{bmatrix} \boldsymbol{M}_t \\ \mathcal{P}_t^{\mathcal{L}} \end{bmatrix}}_{\mathbf{X}_t} = \underbrace{\begin{bmatrix} \boldsymbol{\gamma}_0 \\ 0 \end{bmatrix}}_{\boldsymbol{\Gamma}_0} + \underbrace{\begin{bmatrix} & \boldsymbol{\gamma}_1 \\ \boldsymbol{I}_{\mathcal{L}} & \mathbf{0}_{\mathcal{L}\times(\mathcal{N}-\mathcal{L})} \end{bmatrix}}_{\boldsymbol{\Gamma}_1} \mathcal{P}_t^{\mathcal{N}}. \tag{28}$$

Once $\boldsymbol{\Gamma}_1$ is available, we can find the portfolio weights $\mathbf{W}$ of our state variables $\mathbf{X}_t$ as $\mathbf{X}_t = \mathbf{W}\boldsymbol{y}_t$, where $\mathbf{W} = \boldsymbol{\Gamma}_1 \mathbf{W}^{\mathcal{N}}$.[7] These portfolio weights are thus used to perform a rotation from the latent states $\mathbf{Z}_t$ to the observed state vectors $\mathbf{X}_t$, using the transformations given by the relations in (27).

In the estimation of our macro-finance term structure model, we work with demeaned data, so that $\boldsymbol{\gamma}_0 = \mathbf{0}_{2\times 1}$ and $\boldsymbol{\gamma}_1$ is a $(2 \times \mathcal{N})$ dimensional vector. Our choice of $\mathcal{P}_t^{\mathcal{N}}$ is of dimension (3×1) and contains the yields with maturities of one, four and twenty quarters. The $\mathcal{P}_t^{\mathcal{L}}$ term in our model will be a scalar and consist of the short rate r_t. In the next section we introduce the New Keynesian policy model for the state vector. As we let the aggregate demand and supply shocks follow an AR(1) process, the solved reduced form model structure will be that of a second order VAR process, ie., a VAR(2). We follow Joslin et al. (2013a) and let the lags of the macroeconomic variables have no influence on time t interest rates, which we enforce by having the state vector under $\mathbb{Q}$ contain $\mathbf{X}_t = [\pi_t; y_t; r_t]$, while under the physical measure we have $\mathbf{X}_t = [\pi_t; \pi_{t-1}; y_t; y_{t-1}; r_t]$.

2.4. State dynamics follow a New Keynesian model

We use a New Keynesian Policy Model (NKPM) specification similar to Cho and Moreno (2006) and Buncic and Melecky (2008) to describe the state dynamics in our model. That is, the state variables satisfy the following backward and forward looking NKPM relations:

$$\pi_t = \theta_\pi \mathbb{E}_t[\pi_{t+1}] + (1-\theta_\pi)\pi_{t-1} + \alpha x_t + \varepsilon_t^\pi, \tag{29a}$$

$$x_t = \theta_x \mathbb{E}_t[x_{t+1}] + (1-\theta_x)x_{t-1} - \beta(r_t - \mathbb{E}_t[\pi_{t+1}]) + \varepsilon_t^x, \tag{29b}$$

$$r_t = \theta_r r_{t-1} + (1-\theta_r)(\kappa_\pi \mathbb{E}_t[\pi_{t+1}] + \kappa_x x_t) + \varepsilon_t^r, \tag{29c}$$

where π_t, x_t and r_t are inflation, the output gap and the short rate, respectively. The aggregate supply and demand shocks ε_t^π and ε_t^x in (29) follow a first order autoregressive process of the form:

$$\varepsilon_t^\pi = \rho_\pi \varepsilon_{t-1}^\pi + \nu_t^\pi, \tag{30a}$$

$$\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \nu_t^x, \tag{30b}$$

where ν_t^i are $i.i.d. \sim \mathrm{N}(0, \sigma_i^2)$ for all $i = \{\pi, x\}$. The monetary policy shock ε_t^r is assumed to be $i.i.d. \sim \mathrm{N}(0, \sigma_r^2)$.

The relation in (29a) defines a standard *hybrid* Phillips Curve specification that relates current inflation to expected future and past inflation, as well as the current output gap. The classic Phillips Curve defines a relationship between unemployment and inflation only. Before the 1970s, unemployment and inflation moved inversely to each other, but the occurrence of simultaneously high levels of unemployment and inflation in the 1970s lead to a break down of this relationship. This breakdown is commonly attributed to the fact that private agents began to anticipate monetary policy changes and adapted their behavior accordingly (ie., the Lucas critique). The expectations term in (29a) incorporates the fact that private agents are forward looking (see also Sims (2009)).

Eq. (29b) postulates that the output gap depends on its expected value one period ahead and its lagged value, where the relative impact is determined by the size of θ_x. The forward looking term is due to households' intertemporal optimising behaviour and the lagged term arises as a result of external consumption habit formation, or due to a costly adjustment of the capital stock (see, for instance, Clarida et al. (2002) for more details). When private agents expect higher output in the future, they raise current consumption and current output. The real interest rate affects output through its impact on savings and investment decisions.

We use a standard forward-looking version of the Taylor rule in (29c) (see Clarida et al. (2000) for more details) to close the model. The θ_r parameter in (29c) balances the importance of policy inertia, which are widely documented in U.S. data, and forward looking (or expected) inflation. As is commonly assumed in these type of models and contrary to the specification of the aggregate demand and supply shocks in (30), monetary policy shocks are assumed to be uncorrelated over time.

[7] Note that if the first ℓ principal components are used to construct the portfolio of yields, the weight vector $\mathbf{W}$ is the set of eigenvectors corresponding to the ℓ largest eigenvalues of the covariance matrix of the yields $\boldsymbol{y}_t$.

2.5. Solving the rational expectations equilibrium

We follow Rudebusch and Wu (2008) and many others in the literature, and solve the equations of the NKPM in (29) using the solution algorithm of Sims (2001).[8] To briefly outline the solution approach here, we re-formulate the system of equations in (29) in a standard compact matrix form as:

$$\mathbf{G}_0\mathcal{Y}_t = \mathbf{G}_1\mathcal{Y}_{t-1} + \mathbf{\Psi}\mathbf{z}_t + \mathbf{\Pi}\boldsymbol{\eta}_t, \tag{31}$$

where

$$\mathcal{Y}_t = [\pi_t; \pi_{t-1}; x_t; x_{t-1}; r_t; \mathbb{E}_t[\pi_{t+1}]; \mathbb{E}_t[x_{t+1}]; \mathbb{E}_{t-1}[\pi_t]; E_{t-1}[x_t]] \tag{32}$$

is the (9×1) stacked vector of endogenous variables, $\mathbf{G}_0$, $\mathbf{G}_1$, $\mathbf{\Psi}$ and $\mathbf{\Pi}$ are conformable parameter matrices defined exactly in the Appendix, $\mathbf{z}_t$ are exogenous (or fundamental) shocks with the property that $\mathbb{E}_{t-1}[\mathbf{z}_t] = \mathbf{0}$ and $\mathbb{E}_{t-1}[\mathbf{z}_t\mathbf{z}_t'] = \boldsymbol{R}_{zz}$. The endogenous (non-fundamental) shocks $\boldsymbol{\eta}_t$ have the interpretation of expectational errors, and are defined as $\boldsymbol{\eta}_t = \mathcal{Y}_t - \mathbb{E}_{t-1}[\mathcal{Y}_t]$.

Following Sims (2001), we re-write the parameter matrices in (31) in a form where the first block contains all equations with the stable roots, and the second block, all the equations with the unstable roots. This is achieved by applying the QZ decomposition to $\{\mathbf{G}_0, \mathbf{G}_1\}$, that is, we apply the following factorisation:

$$\mathbf{G}_0 = \mathcal{Q}\mathcal{S}\mathcal{Z}', \quad \text{and} \quad \mathbf{G}_1 = \mathcal{Q}\mathcal{T}\mathcal{Z}', \tag{33}$$

where $\mathcal{Q}$ and $\mathcal{Z}$ are orthonormal matrices, and $\mathcal{S}$ and $\mathcal{T}$ are upper triangular with the generalized eigenvalues of $\{\mathbf{G}_0, \mathbf{G}_1\}$ on the diagonal. The first block contains all equations where the ratio of the absolute value of the eigenvalues of $\mathbf{G}_1$ to the eigenvalues of $\mathbf{G}_0$ are smaller than one, the second block contains all equations where the ratio is larger or equal to one. By eliminating the influence of all unstable roots, the expectational errors can be solved for and the solution can be represented in a compact VAR(1) of the form:

$$\mathcal{Y}_t = \boldsymbol{S}_1\mathcal{Y}_{t-1} + \boldsymbol{S}_2\mathbf{z}_t, \tag{34}$$

where in our model $\boldsymbol{S}_1$ is of dimension (9×9) and $\boldsymbol{S}_2$ is (9×3).

Note that in the analysis that follows, we use $\mathbf{X}_t = [\pi_t; \pi_{t-1}; y_t; y_{t-1}; r_t]$ as the state vector under the physical measure, since the conditional expectations terms in $\mathcal{Y}_t$ can be expressed as linear combinations of the state variables in $\mathbf{X}_t$ once the model is solved. This means then that the law of motion for the state vector is a VAR(1) as before:

$$\mathbf{X}_t = \boldsymbol{\mu} + \mathbf{\Phi}\mathbf{X}_{t-1} + \mathbf{\Sigma}\boldsymbol{\epsilon}_t, \tag{35}$$

however with $\mathbf{\Phi}$ the upper left (5×5) component of $\boldsymbol{S}_1$ and $\mathbf{\Sigma}$ the upper (5×3) part of $\boldsymbol{R}_{zz}$.

3. Data and estimation approach

3.1. Data

We estimate our model on quarterly data, covering a sample period from 1962:Q1 to 2014:Q2. All term structure data are zero-coupon yields taken from the Gürkaynak et al. (2007) (henceforth GSW) database. We construct quarterly interest rates from daily data, by using quarterly averages of the daily rates. We follow Rudebusch and Svennson (2002); Ang et al. (2011) and many others, and construct the output gap as:

$$x_t = \frac{\text{GDP}_t - \text{GDP}_t^*}{\text{GDP}_t^*}, \tag{36}$$

where GDP_t and GDP_t^* are real gross domestic product and real potential output, respectively. The potential and real output measures are published by the Congressional Budget Office (CBO) and the U.S. Department of Commerce, Bureau of Economic Analysis (BEA), respectively. Both, the BEA and CBO data are available from the website of the Federal Reserve Bank of St. Louis (the url of the FRED2 database is https://research.stlouisfed.org/fred2/). Both time series are produced using chained 2009 dollars and are in annual terms. Our inflation series is the year-on-year GDP deflator expressed as a continuously compounded growth rate. To make the data comparable to the interest rate data, that is, to quarterly units, the inflation and the output gap measures are divided by four. Also, for estimation, all data are demeaned as in Rudebusch and Wu (2008) and Buncic and Melecky (2008). Time series plots of the three macroeconomic variables are shown in Fig. 1.

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[8] That is, we use the `gensys.m` Matlab function, freely available from http://sims.princeton.edu/yftp/gensys/ to solve the model. A comprehensive overview of the algorithm of Sims (2001) can be found in Farmer et al. (2015).

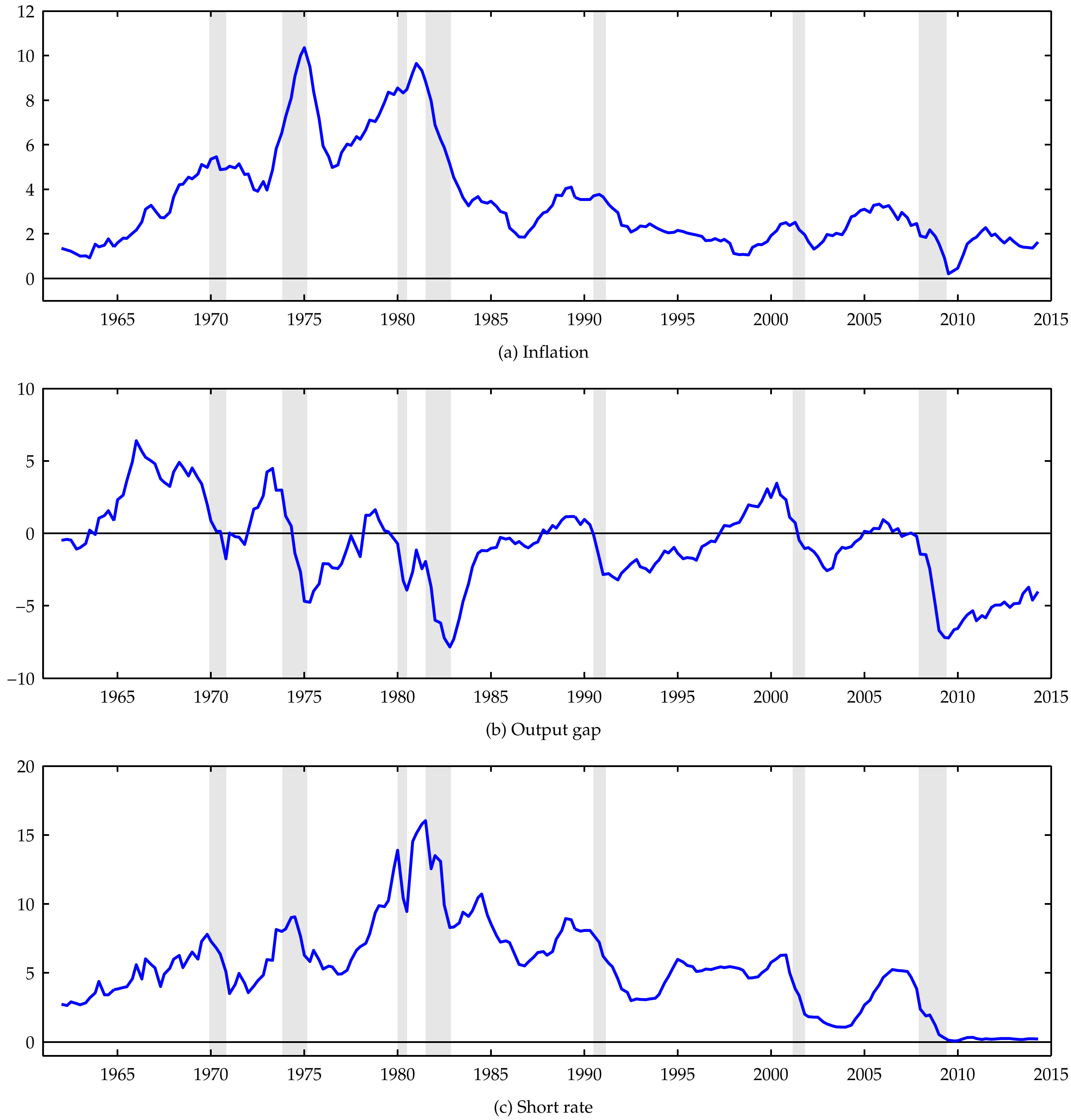


Fig. 1. Time series of inflation, the output gap and the short rate: Inflation is the year-on-year change in the GDP deflator. The output gap is the difference between real GDP and real potential GDP. The short rate is the one quarter rate taken from the Gürkaynak et al. (2007) database. All data are quarterly and figures show annualized percentage terms. The sample period is from 1962:Q1 to 2014:Q2. Shaded areas indicate NBER recession periods.

3.2. Full information maximum likelihood estimation

Let $\boldsymbol{y}_t$ again denote the $J-$dimensional stacked vector of all observable bond yields. In what follows, we will denote by $\boldsymbol{y}_t^e$ all bond yields that are measured with error. Both, the macroeconomic factors as well as the short rate are assumed to be observed without measurement error.[9] Yields with maturities $n \in \{2, 4, 12, 20\}$, ie., $\boldsymbol{y}_t^e = \left[y_t^{(2)}; y_t^{(4)}; y_t^{(12)}; y_t^{(20)}\right]$, which

[9] As in Bauer and Rudebusch (2014b), we do not allow for measurement errors in the macroeconomic factors, because in that case, *'the likelihood function largely gives up on fitting the observed macroeconomic factors in favor of more accurate pricing of bonds'* (Joslin et al. (2013b)).

Table 1
Maximum likelihood estimates of the NKPM structural parameters.

Equation	Parameter	Estimate	Standard error
Inflation dynamics:	θ_π	0.5017***	0.0131
$\pi_t = \theta_\pi \mathbb{E}_t[\pi_{t+1}] + (1-\theta_\pi)\pi_{t-1} + \alpha x_t + \varepsilon_t^\pi$	α	0.0027*	0.0016
$\varepsilon_t^\pi = \rho_\pi \varepsilon_{t-1}^\pi + \nu_t^\pi$,	ρ_π	0.5209***	0.0662
Output dynamics	θ_x	0.5128***	0.0107
$x_t = \theta_x \mathbb{E}_t[x_{t+1}] + (1-\theta_x)x_{t-1} - \beta(r_t - \mathbb{E}_t[\pi_{t+1}]) + \varepsilon_t^x$	β	0.0009	0.0013
$\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \nu_t^x$,	ρ_x	0.3157***	0.0739
Short rate dynamics	θ_r	0.8995***	0.0232
$r_t = \theta_r r_{t-1} + (1-\theta_r)(\kappa_\pi \mathbb{E}_t[\pi_{t+1}] + \kappa_x x_t) + \varepsilon_t^r$	κ_π	1.3274***	0.2344
	κ_x	0.8170***	0.2335
Standard deviation of $\nu_t^\pi, \nu_t^x, \varepsilon_t^r$ ($\times$ 400)	σ_π	0.1563	0.0079
	σ_x	0.3923	0.0201
	σ_r	0.7622	0.0373

This table reports maximum likelihood estimates of the structural parameters of the New Keynesian Policy Model (NKPM) as defined in (29). The first column shows the estimating equations, the second column the parameter of interest, the third the estimates and the forth the standard error. The standard errors are based on numerical evaluation of the Hessian matrix. Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

correspond to maturities of 0.5, 1, 3, and 5 years, are measured with errors as:

$$\boldsymbol{y}_t^e = \mathbf{a} + \mathbf{b}\mathbf{X}_t + \boldsymbol{e}_t, \tag{37}$$

where the $[(J-1)\times 1]$ dimensional vector of measurement errors $\boldsymbol{e}_t$ is assumed to be *i.i.d.* multivarite normal distributed with a diagonal variance/covariance matrix $\boldsymbol{\Omega}_e$, and $\mathbf{a}$ and $\mathbf{b}$ are the stacked vector form of a_n and $\boldsymbol{b}_n$ in (18).

Given these assumptions, the likelihood function can be constructed by factoring the joint conditional density as:

$$f(\boldsymbol{y}_t^e, \mathbf{X}_t|\mathbf{X}_{t-1}; \boldsymbol{\Theta}) = f(\boldsymbol{y}_t^e|\mathbf{X}_t; \boldsymbol{\Theta}) \times f(\mathbf{X}_t|\mathbf{X}_{t-1}; \boldsymbol{\Theta}) \tag{38}$$

$$= f(\boldsymbol{y}_t^e|\mathbf{X}_t; \boldsymbol{\lambda}^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1\boldsymbol{\Sigma}, \boldsymbol{\Omega}_e) \times f(\mathbf{X}_t|\mathbf{X}_{t-1}; \boldsymbol{\Phi}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{39}$$

where $f(\mathbf{X}_t|\mathbf{X}_{t-1}; \boldsymbol{\Phi}, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ follows from the definition of the state dynamics defined in (35) and $f(\boldsymbol{y}_t^e|\mathbf{X}_t; \boldsymbol{\lambda}^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \boldsymbol{\Sigma}, \boldsymbol{\Omega}_e)$ is from (37). The parameter set $\boldsymbol{\Theta} = \{\boldsymbol{\lambda}^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1, \boldsymbol{\Phi}, \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{\Omega}_e\}$ is estimated by maximizing the log-likelihood function, that is, by solving:

$$\arg\max_{\boldsymbol{\Theta}} \sum_{t=3}^{T} \mathscr{L}_t(\boldsymbol{\Theta}) \tag{40}$$

where

$$\mathscr{L}_t(\boldsymbol{\Theta}) = -\frac{(J-1)}{2}\ln(2\pi) - \frac{(J-1)}{2}\ln(\det(\boldsymbol{\Omega}_e)) - \frac{1}{2}\boldsymbol{e}_t'\boldsymbol{\Omega}_e^{-1}\boldsymbol{e}_t \tag{41}$$

$$-\frac{3}{2}\ln(2\pi) - \frac{1}{2}\ln(\det(\boldsymbol{\Omega})) - \frac{1}{2}(\mathbf{X}_t - \boldsymbol{\mu} - \boldsymbol{\Phi}\mathbf{X}_{t-1})'\boldsymbol{\Omega}^{-1}(\mathbf{X}_t - \boldsymbol{\mu} - \boldsymbol{\Phi}\mathbf{X}_{t-1}), \tag{42}$$

where $\boldsymbol{\Omega} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}'$, and $\boldsymbol{e}_t = \boldsymbol{y}_t^e - \mathbf{a} - \mathbf{b}\mathbf{X}_t$ is as defined from (37).

Note that we find $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}$ by maximising with respect to the structural parameters from the inflation, output and the short rate equation $\boldsymbol{\theta} = [\theta_\pi; \theta_x; \theta_r; \alpha; \beta; \kappa_\pi; \kappa_x; \sigma_x^2; \sigma_\pi^2; \sigma_r^2]$. Given these structural parameters, the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}$ have to be calculated at every iteration of the maximization algorithm. We calculated asymptotic standard errors by evaluating the inverse of the negative Hessian at the convergence points of the log-likelihood function, where the Hessian matrix is compute numerically.[10]

4. Empirical results

4.1. Parameter estimates

In Table 1 we report estimates of the parameters of the NKPM in (29). The estimated weight on forward looking or expected inflation (θ_π) is approximately 0.50. Cho and Moreno (2006) find a forward looking parameter estimate of 0.55, while Buncic and Melecky (2008) compute a parameter estimate somewhat higher at 0.65 for their U.S. block. The estimate reported in Rudebusch and Wu (2008) is 0.074 for monthly data, which, when converted to a quarterly frequency, implies a somewhat smaller coefficient of around 0.22 at a quarterly horizon. The estimate of Bekaert et al. (2010) is around 0.61.

[10] We use parts of the Matlab code that implements the Joslin et al. (2011) normalization scheme from Singleton's webpage http://web.stanford.edu/~kenneths/jsz_code_posted.zip.

Table 2
Maximum likelihood estimates of $\mathbb{Q}$ parameters.

Parameter	Estimate	Standard errors
$\lambda_1^{\mathbb{Q}}$	0.9998***	0.0012
$\lambda_2^{\mathbb{Q}}$	0.9777***	0.0790
$\lambda_3^{\mathbb{Q}}$	0.3295**	0.1521
$\sigma_u(\times 400)$	0.6470***	0.0160

This table reports Maximum Likelihood estimates of the $\mathbb{Q}$ parameters ($\boldsymbol{\Psi}^{\mathbb{Q}}$) as defined in 22. Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

Table 3
Maximum likelihood estimates of macro spanning parameters.

Parameter	$y_t^{(1)}$	$y_t^{(4)}$	$y_t^{(20)}$
π_t	42.12***	−56.45***	16.51***
(std.error)	(4.85)	(3.96)	(4.10)
y_t	−28.50***	46.13***	−19.92***
(std.error)	(5.34)	(5.11)	(3.59)

This table reports the Maximum Likelihood estimates of the macro spanning parameters $\boldsymbol{\gamma}_1$ in (28). Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

Overall, the magnitude of our point estimate is reasonable and in line with values found in the literature. Our estimate of the Phillips Curve parameter α of 0.0027 is rather low and only marginally significant. This estimate is also in line with other findings in the literature. For instance, the estimate in Cho and Moreno (2006) (see the λ parameter estimate in Table 4 of their paper) is 0.0023. Bekaert et al. (2010) have argued that the failure to obtain a more reasonably sized and significant estimate of this parameter is likely due to measurement errors in the macroeconomic variables. Inertia in the aggregate supply shock are estimated at 0.53.

The parameter estimates describing the output gap dynamics are also in line with estimates found in the existing literature. Our estimate of the parameter on the forward looking expectation term in the output gap equation is 0.5128. Buncic and Melecky (2008) and Cho and Moreno (2006) obtain comparable estimates in the 0.48 to 0.54 range, while the one found in Bekaert et al. (2010) is marginally lower than ours at 0.42. Interestingly, the study by Rudebusch and Wu (2008) finds a more benign impact from the forward looking term on the output gap. Our estimate of the β parameter, which determines the role of monetary policy in the output gap evolution, is small in magnitude and further not different from 0 statistically. As was the case with the inflation equation, this result is consistent with previous studies, which have found it difficult to obtain significant estimates of β. As an example, the range of values found in Cho and Moreno (2006) and Buncic and Melecky (2008) is between 0.0017 to 0.0065. Our estimate of the parameter capturing aggregate supply shock dynamics of 0.3157 is very close to the value reported in Cho and Moreno (2006) of 0.3555.

Looking over the Taylor rule parameter estimates, we can see that our θ_r estimate of 0.8995 is closer to the upper range of values reported in Cho and Moreno (2006). Buncic and Melecky (2008) report a point estimate of 0.8179, which is somewhat lower, with their 95% Bayesian posterior confidence intervals covering the range from 0.76 to 0.87. The two parameters that capture the responsiveness of the short rate r_t to (future) expected inflation and the (current) output gap, that is, κ_π and κ_x are estimated at 1.3274 and 0.8170, respectively. Comparing these values to other estimates in the literature, we can see that the κ_π estimate is somewhat smaller than the lower value found in Cho and Moreno (2006) and Buncic and Melecky (2008) of around 1.6, while the κ_x estimate falls in the middle of the range of values reported in Cho and Moreno (2006). Overall, we can conclude here that our NKPM parameter estimates are in the range of values found in other studies in the literature.

For completeness of our estimation results, we also report the maximum likelihood estimates of the $\mathbb{Q}$ parameters in Tables 2 and 3. To the best of our knowledge, there exists no other study that reports estimates of the $\boldsymbol{\gamma}_1$ parameter values within an MTSM which we could use as a benchmark to compare our results to. Joslin et al. (2013b) do not report estimation results and, although Bauer and Rudebusch (2014b) also employ the JLS normalization scheme, they do not report their estimates of $\boldsymbol{\gamma}_1$ in the paper either. We thus simply report our results without further discussing the magnitude and the relation to other values found in the literature.

In the last row of Table 2, we report the average of the estimated annualised standard deviation of the bond yield measurement errors, which is $\hat{\sigma}_e \times 400 = 0.65$. Including more yield curve factors could have the beneficial effect of reducing

Table 4
Comparison of implied theoretical and empirical moments of the data.

Moments	Sample				Theoretical			
	π_t	x_t	r_t	$y_t^{(40)}$	π_t	x_t	r_t	$y_t^{(40)}$
σ	2.2194	2.8755	3.2254	2.5847	2.6855	2.8193	4.3313	3.5875
ACF(1)	0.9820	0.9589	0.9606	0.9782	0.9905	0.9596	0.9817	0.9775
ACF(2)	0.9478	0.8956	0.9103	0.9501	0.9705	0.8969	0.9611	0.9531
ACF(3)	0.9030	0.8173	0.8690	0.9237	0.9448	0.8296	0.9388	0.9274
ACF(4)	0.8509	0.7335	0.8191	0.8941	0.9158	0.7638	0.9148	0.9005
Corr(x_t, π_t)	0.0026				0.2694			
Corr(x_t, r_t)	0.7207				0.8145			
Corr(r_t, π_t)	0.1954				0.3431			

This table shows the comparison between the model implied theoretical moments and the empirical (sample) moments of the data. The unconditional standard deviation is denoted by σ, ACF(·) denotes the value of the autocorrelation function and Corr is the standard (Pearson) correlation.

the size of the measurement errors.[11] Nevertheless, we decided to keep our model *'small'*, thereby avoiding the need to add another factor without having a clear economic interpretation of it. We leave this as a potential future research topic, that is, how the state vector in our small NKPM could be extended in an economically meaningful way, so that the slope of the yield curve is better captured.

4.2. Assessing the fit of the model

Inspecting the residuals of the state equations gives us an idea of how well the New Keynesian policy model reflects the dynamics of the data. We use standard residual mis-specification tests to assess the goodness-of-fit of the model. A visual inspection of the histograms of the ν_t^x, ν_t^π and ε_t^r residuals shows that they are *'Gaussian'* looking in the sense that they are symmetrically distributed, with a normal level of *'fatness'* of the tails.[12] There are a handful of observations in the residuals of the short rate equation that are causing a mild right skew in the distribution, with the distribution showing further a higher peak around 0, which is a well known trait of interest rate data. Ljung-Box tests for first order serial dependence in the residuals indicate that there is no evidence of serial correlation in the inflation and output gap residuals ν_t^π and ν_t^x, with $p-$values of around 0.45.

Monetary policy shocks, on the other hand, appear to be mildly correlated, having a first order autocorrelation coefficient of 0.16. The $p-$value of the corresponding Ljung-Box test is 0.0156, which one could consider as weakly significant, ie., at the 5% level. The dynamics of the short rate thus do not appear to be fully captured by our relation in (29c). As is extensively discussed in the literature, the post-Volcker policy rule and the pre-Volcker counterpart should ideally be treated as two different regimes (see, for instance, the discussion in Clarida et al. (2000)), where only the former is consistent with an optimal monetary policy strategy. This is commonly viewed as the source of the serial correlation found in the residuals of Eq. (29c). Although it would be possible to estimate a regime-switching model which would account for this change in the policy environment, as is done in the structural MTSM of Bikbov and Chernov (2013), for instance, it would make estimation of the model considerable more complicated.[13] We therefore do not pursue this strategy here. Moreover, the level of serial correlation in the monetary policy shocks is rather low, thus only creating a marginal distortion.[14]

As a final and fairly stringent assessment of the fit of the model to the data, we compare the implied theoretical moments from the MTSM to the empirical counterparts from the data. Similar approaches to assess the fit of models have been used in Buncic and Melecky (2008) and Cho and Moreno (2006) for NKPMs, but also for dynamic term structure models in Dai and Singleton (2002). The results from this comparison are shown in Table 4. Since we work with demeaned data, we focus on a comparison of second moments, cross correlations and autocorrelations. To compute the theoretical moments,

[11] The estimated measurement errors of yields are highly correlated with the slope of the yield curve (the estimated measurement errors of the 40 quarter yield has a correlation of 0.78 with the slope of the yield curve defined as the difference of the 40 quarter yield and the one quarter yield). The inclusion of a slope factor would thus likely decrease the measurement errors.

[12] We do not show plots of the histograms to converse space, but these are available from the authors upon request.

[13] Alternatively, one could also let the parameters evolve slowly over time, as is done for instance, in Primiceri (2005), or with a some selection and/or model averaging as is done in Buncic and Moretto (2015) and Buncic and Piras (2016b) for financial variables to account for model uncertainty as well, or specify parametric non-linear functions such as smooth transition models for interest rates, see for instance Sarno et al. (2003) , but also the papers by Buncic (2012) and Buncic (2016a) to understand some of the drawbacks with this approach.

[14] We should also note here, that, due to one referees request, we have re-estimated the entire model over a period that excludes the global financial crisis period, that is, where the sample ends in 2006:Q4. These results are reported in Appendix A.5. From these results we can see that: (a) our re-estimated MTSM provides a better fit to the empirical yield curve data (difference between the empirical and model implied standard deviation of the 10 year yield is now $3.0589 - 2.4894 = 0.5695$ as opposed to $3.5875 - 2.5847 = 1.0028$), (b) the empirical moments of the data are better matched by the model implied ones, capturing even the switch in the correlations between inflation and output, and (c) the policy implications are rather robust to the change in the sample period, generating similar impulse response patterns and variance decompositions. We thus deem the model to be reasonable in reproducing the properties of the data, even when different sample periods are considered.

we simulate 500, 000 data points from our estimated MTSM and compute the *'sample'* moments from this simulated series as population moment counterparts.

As can be seen from the comparison shown in Table 4, particularly the autocorrelation (ACF) structure in all variables of interest, as well as the variation, captured by the standard deviation σ in the output gap x_t from the theoretical model match the sample data rather well. The theoretical model produces a somewhat larger variation in the inflation series, as well as the short and long horizon interest rates. The model implied cross-correlation between the short rate and the output gap, and the short rate and the inflation series also match the data rather well. One aspect where the theoretical model does not match the data well is the correlation between the output gap and inflation. In the data, this cross-correlation is close to zero. This is also evident from the small and insignificant estimates of the β and α parameters in (29). The theoretical model produces a fairly sizable correlation of around 27%.[15]

4.3. Economic implications of the model

Having inspected the empirical fit of our model to the data, we proceed to investigate its economic implications. Our structural macroeconomic model readily allows us to compute impulse response functions (IRFs) and further to construct forecast error variance decompositions (FEVDs) from the model to measure the impact of aggregate supply, demand and monetary policy shocks on variables of interest in the MTSM.

4.3.1. Analysis of shocks: IRFs and FEVDs

In Fig. 2 we show the impulse responses of bond yields at the 1 quarter (short rate) and 40 quarters (10 year) horizon, as well as their implied risk premia, using a *'one unit'* increase (or shock) in each of the three structural errors of the model as the *'magnitude'* of the shock. Following Cho and Moreno (2006); Buncic and Melecky (2008) and many others, we interpret the inflation shock ε_t^{π} as a *'cost push shock'*, which makes real wages deviate from their equilibrium value. The shock ε_t^{x} in the output gap equation is interpreted as an *'aggregate demand shock'*, while the ε_t^{r} shock in the short rate equation is interpreted as a *'monetary policy shock'*.

From the left column of Fig. 2 we can see that an inflation shock has initially a fairly low immediate impact on the yield curve, with a response close to zero in the quarter when the shock is realised. Interest rates are only slowly responding to the shock. This is due to monetary policy inertia, which, captured by the θ_r parameter in our MTSM, are estimated to be at 0.9. Rudebusch and Wu (2008) allow for serial correlation in the shocks to their short rate equation. The estimate of θ_r they obtain is thus lower, which results in a more instantaneous response of interest rates to macroeconomic shocks. Due to the high degree of monetary policy inertia in our model and the size of the κ_π estimate being greater than one, the short rate responds by more than one to the unit inflation shock, which, nevertheless occurs only gradually after about five quarters. The peak of the response of 1.5 is realised approximately after 14 quarters. High monetary policy inertia also lead to a long-lived response pattern, decreasing only gradually over time.

From the plot in the second row of the left column of Fig. 2, we can see that a (one unit) shock to aggregate demand (ε_t^{x}) induces an overall response in the yields of the two maturities that is very similar to an inflation shock. As was the case with the inflation shock, this response pattern is again influenced by monetary policy inertia. Due to the κ_x estimate being less then unity, the impulse responses stay below unity at all horizons. Overall, it is evident from these results that they correspond closely to our understanding of how standard Taylor rule dynamics work through an economic system, with the central bank putting more emphasis (or weight) on inflation shocks than on output shocks in their objective function.

The responses of the interest rates to a monetary policy shock shown in the Fig. 2 economic intuition. When the central bank increases the short rate by one unit without responding to inflation or the output gap, the whole term structure shifts almost one-to-one upward. This reflects the role of the short rate as a *'level'* factor in our model. The non-linear influence of the short rate on the longer horizon yield leads to an initially milder effect on impact, which, approximately 12 quarters thereafter has the same influence as on the short rate.

The right column of Figure 2 shows the effect of the various macroeconomic shocks on the term premium. We draw the 40 quarter (10 year) as well as the 1 quarter term premium in the plots for consistency, although by definition, the 1 quarter term premium is equal to zero, and hence the impulse responses to all shocks are by definition equal to zero. Consistent with the results reported in Ludvingson and Ng (2009), we find a decreasing term premium in response to positive inflation and output shocks. This is due to the pro-cyclical behaviour of inflation and output, and the strongly counter-cyclical

[15] Note here, that it is extremely difficult to get an accurate match between the theoretical moments an MTSM generates and the empirical data. We report this comparison to provide some additional information to the reader in terms of what the model can replicate, and what it has difficulties in replicating. As a whole, we see this match to be reasonable. We would also like to point our here that we do not need the negative correlation between real output and inflation in our model to generate a positive term premium as in Piazzesi and Schneider (2006). Piazzesi and Schneider (2006) put tight theoretical restrictions on the distribution of the pricing kernel, but leave the distribution of the macro variables unrestricted. Under these tight restrictions, it is difficult to generate a positive term premium that exhibits economically meaningful variation. In our paper, on the other hand, we put restrictions on the distribution of the macroeconomic variables and the short-term interest rate as implied by the New-Keynesian policy model, and do not put theoretical restrictions on the distribution of the pricing kernel (see also Rudebusch and Wu (2008) on this). This allows for a more flexible parameterization of the model for the longer-term interest rates and therefore allows also for the generation of a positive term-premium that exhibits considerable variation. The key difference to Rudebusch and Wu (2008) is that we model the link between the macroeconomic variables and the longer-term yields as in Joslin et al. (2013b). We would like to thank an anonymous referee for bringing this point to our attention.

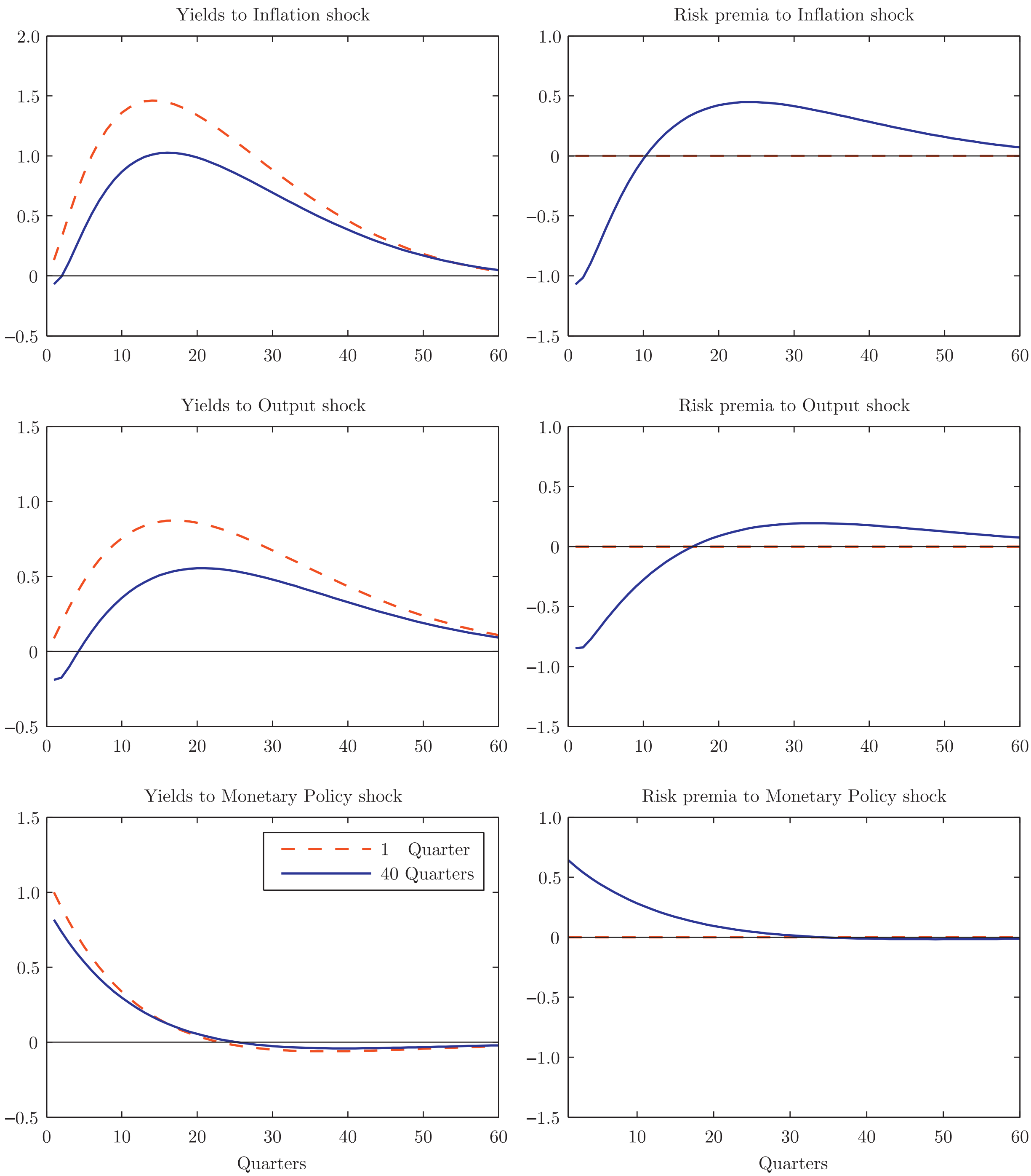


Fig. 2. Impulse response functions of yields and risk premia at the 1 quarter and 40 quarter horizon to shocks in inflation, output, and monetary policy. Shocks are normalized to one-unit shocks.

behaviour of the term premium. Positive shocks to inflation decrease the term premium instantaneously by slightly more than one-to-one, while positive shocks to the output gap lead to a slightly less than one-to-one decrease, which is again due to the different magnitudes in the Taylor rule coefficients (that is, κ_π and κ_x). As market participants anticipate that the central bank will increase the short rate in response to improving economic conditions and rising inflation, there is an instantaneous increase in the expectations hypothesis term of interest rates. In total, the two effects almost offset each other initially. The term premium response reverts to zero relatively quickly, with a half life of about four quarters. Nevertheless, the effect of the expectations hypothesis term is long lasting and reverts back to zero only slowly.

Table 5
Variance decompositions.

Short end	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0028	0.0402	0.2430	0.2319
Output	0.0072	0.0808	0.5095	0.6180
Short rate	0.9900	0.8790	0.2475	0.1501
Middle	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0000	0.0152	0.2439	0.2384
Output	0.0073	0.0119	0.4224	0.5703
Short rate	0.9927	0.9729	0.3337	0.1913
Long end	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0011	0.0062	0.2464	0.2469
Output	0.0490	0.0355	0.3503	0.5303
Short rate	0.9499	0.9583	0.4033	0.2228

This table shows variance decompositions, that is, the contribution of each factor to the h-step ahead forecast error variance of the short end, middle end and long end of the yield curve. Each numerical entry in the three separate (3 × 4) blocks gives the percentage value of the $h-$step ahead forecast error variance explained by inflation, output, and the short rate. For instance, in the Short end block, 99% of the 1−step ahead forecast error variance at the short end of the yield curve is explained by the short rate itself.

We proceed in our analysis by constructing forecast error variance decompositions (FEVDs) to gauge the relative contribution of the various macroeconomic factors on the variation in the yield curve. These FEVDs are reported in Table 5. The study by Ang and Piazzesi (2003) finds that, depending on the model specification considered, inflation has more (or at least equal) explanatory power than real activity. Since the Taylor rule weight on inflation is larger than on the output gap, it is reasonable to expect that inflation should have a larger explanatory role than the output gap. In our estimation results, however, we find that real activity explains at most forecasting horizons a larger part of the forecast error variance than inflation. The reason for this result is that the conditional standard deviation of the output gap is estimated to be about two times larger than the conditional standard deviation of inflation. This puts more weight on output shocks when they hit the economic system than on inflation shocks, which allow for the possibility of output to have more explanatory power than inflation, despite the Taylor rule weight on inflation being larger than on the output gap. In summary, we can conclude from the FEVD analysis that inflation and the output gap explain a large proportion of the model implied bond yields.[16]

As a final remark, we should stress here that Ang and Piazzesi (2003) find the proportion of the unconditional variance accounted for by their macroeconomic factors to be noticeably decreasing with the maturity of the yields. The highest is observed at the short and middle ends of the yield curve, and the smallest at the long end. We only find a small decrease in the explanatory power of our macroeconomic factors across all maturities, dropping from 85% at the short end to 78% at the long end (see that last column entries in Table 5). Our intuition is that the drop occurs because we only use one yield curve factor (the short rate) in addition to the macroeconomic factors as our state variables. Although the proportion of the variation explained at the long end decreases somewhat, the 78% may not accurately reflect the rather good explanatory power that the macroeconomic state variables have in explaining the variation in longer horizon yields.

4.3.2. Term premia and expectations hypothesis

In Fig. 3 we plot the 10 year (40 quarters) term premium as defined in (5). In Fig. 3 we also superimpose the estimated model implied, as well as the empirical, 10 year bond yields. As can be seen, the model implied yield (green line) and the (empirical) 10 year yield (red line) track one another rather well. One should keep in mind here that the 10 year yield is not included in the set of yields in the calibration of our model. We can thus think of this comparison as a cross-sectional out-of-sample comparison.[17] Comparing our estimates of the term premium from the MTSM to other recent ones by Adrian et al. (2013), we can see that the overall movements match closely. There are, nevertheless, some important differences. Firstly, the term premium estimate of Adrian et al. (2013) is close to, but above, zero around the 2005 to 2006 period, while ours becomes marginally negative. Secondly, during the early 80s (tight monetary policy) recession, our estimate of the term premium is larger and also appears to show a larger response to the recession, as indicated by a steeper upward movement. And thirdly, the term premium estimate of Adrian et al. (2013) turned negative towards the end of 2012, while ours still

[16] We also decompose the forecast error variance of the term premium part of yields into the contribution of each of the shocks. Unconditionally, the results are similar to the ones when decomposing the forecast variance of bond yields. For reasons of brevity and repetitiveness, we do not report these results here.

[17] The correlation between our estimated 10 year term premium and our measure of real activity, ie., the output gap x_t, is −0.83. This negative correlation reflects the counter-cyclical behaviour of investors' risk compensation demands. During economic expansions, when real activity is relatively high, market participants are optimistic, which is reflected in a lower premium for bearing risky assets. Conversely, during economic recession, when real activity is low, investors are more cautious and demand a higher risk premium to buy risky-assets.

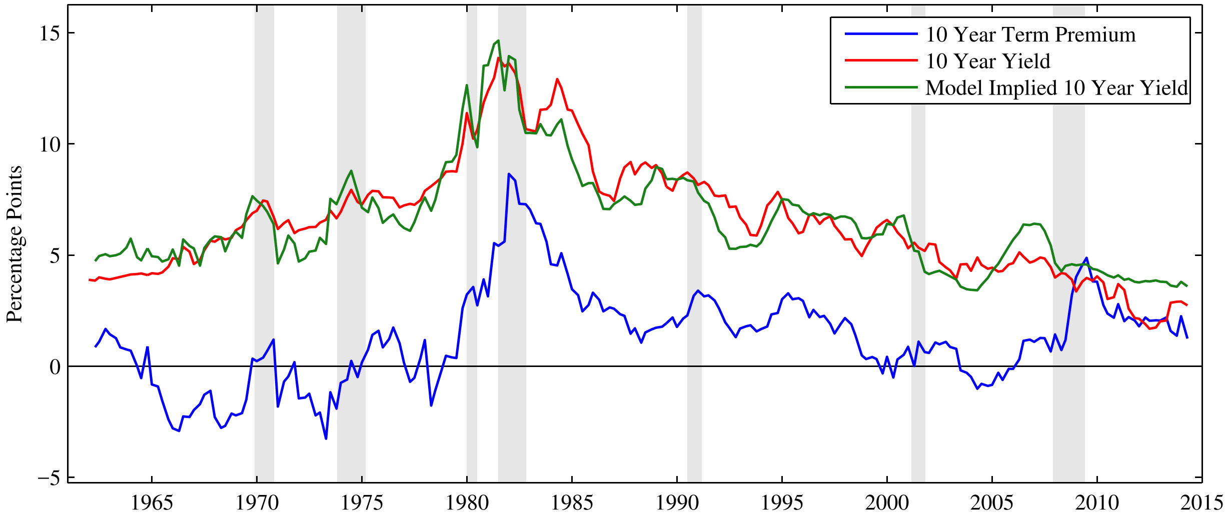


Fig. 3. Term premium and bond yields: This figure shows the term premium component of the ten year spot rate as estimated from our MTSM described in the text (blue line), together with the model implied 10 Year Yield (green line) and \the (empirical) 10 Year Yield ($y_t^{(40)}$) (red line) from the Gürkaynak et al. (2007) database. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

remains positive even towards the end of 2014. Overall, it is also noticeable that our MTSM based term premium seems to be somewhat more volatile, with larger variation across time.[18]

As a last '*test*' of the estimated MTSM, we examine whether it can generate plausible yields and risk premia that are consistent with empirically observed facts of the expectation hypothesis puzzle documented in Campbell and Shiller (1991). This is implemented by computing the two '*diagnostics tests*' frequently referred to as "*linear projection of yields*" test (i) and (ii), or more compactly as, LPY(i) and LPY(ii), proposed by Dai and Singleton (2002), to assess the fit of the model. Before we turn to an assessment of LPY(i) and LPY(ii), recall that under the expectations hypothesis (EH), the yield on an n-period bond is equal to the average of expected future short rates. This means then that the yield on an n-period bond should increase one-to-one when the term spread widens. Campbell and Shiller (1991) conveniently reformulate this prediction from the EH as a regression (henceforth, CS regression) of bond returns $(y_{t+1}^{(n-1)} - y_t^{(n)})$ on the scaled slope of the yield curve $(y_t^{(n)} - r_t)/(n-1)$. That is,

$$(y_{t+1}^{(n-1)} - y_t^{(n)}) = \phi_0 + \phi_n \frac{(y_t^{(n)} - r_t)}{(n-1)} + \epsilon_{t+1}, \tag{43}$$

for all $n = 2, 3, \ldots, J$, where ϵ_{t+1} is a random disturbance term with a zero mean and uncorrelated with $(y_t^{(n)} - r_t)$. If the EH holds, ϕ_n should be close to 1 for all n . In Table 6 we report estimates of ϕ_n computed from our empirical yield curve data for the various maturities that we consider.[19] Consistent with the findings reported in Campbell and Shiller (1991), the estimates of ϕ_n are negative, becoming increasingly so for longer maturity bonds.

Let us now turn to LPY(i) and LPY(ii) of Dai and Singleton (2002), which are plotted in the top and bottom panels of Fig. 4.

LPY(i): Can our estimated MTSM reproduce the observed negative relation between bond returns and the (scaled) slope of the yield curve? To address this questions, we compute the population quantity of $\phi_n, \forall n = 2, 3, \ldots, J$ implied by the MTSM at the maximum likelihood estimates and compare these to the ϕ_n estimated from the data. If this property is well matched by the estimated MTSM, these quantities should follow the same overall pattern. To derive the model implied ϕ_n, we write the regression coefficient ϕ_n in (43) as:

$$\phi_n = \frac{\text{Cov}\,[(y_{t+1}^{(n-1)} - y_t^{(n)}), (y_t^{(n)} - r_t)/(n-1)]}{\text{Var}[(y_t^{(n)} - r_t)/(n-1)]}. \tag{44}$$

[18] Note here also that in the first few quarters of 2009, our term premium estimate was higher then the corresponding long rate, which would imply that market participants were expecting negative future short term (1 quarter) rates. Although this seems rather unusual, there were a number of instances where the 3 month treasury bill rate turned negative. At an international level, short term rates are currently negative in Germany, Sweden as well as in Japan. Since affine term structure models do not put a zero lower bound on the bond yields, this results is consistent with our MTSM (see Bauer and Rudebusch (2014b)).

[19] Note that we use Generalised Least Squares (GLS) to get estimates of ϕ_n. We allow the residuals to follow an AR(1) process and use the estimated AR(1) autocorrelation coefficient to compute the variance/covariance matrix of ϵ_{t+1}, which is then utilized to construct the weighting matrix in the GLS procedure. Following Cochrane-Orcutt estimation, we iterate until convergence, using a convergence criterion of 10^{-6} on the $\hat{\phi}_n$ coefficient, starting the recursions from the OLS estimate.

Table 6
Campbell and Shiller regressions.

Maturity in quarters	Estimate	Standard errors
4	−0.4993	0.3744
12	−1.2028**	0.5759
20	−1.6527**	0.6863
32	−2.2188***	0.8233
40	−2.5818***	0.9130

This table reports the empirical estimates of Campbell and Shiller regression coefficient ϕ_n from the regression $(y_{t+1}^{(n-1)} - y_t^{(n)}) = \phi_0 + \phi_n[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$. This regression was estimated with Generalized Least Squares (GLS), where the residuals follow first order autoregressive processes. Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

and compute these analytically.[20] Additionally, to examine the small-sample analogues of the population ϕ_n from the model, we simulate 10, 000 samples of length 210 (the length of our empirical data) from the model at the MLE estimates. For each sample, the projection coefficient ϕ_n is computed analytically. The mean of these estimates across the 10, 000 samples is displayed in Panel (a) of Fig. 4 as "Model-implied (MC)" and the 90th and 10th percentiles are displayed as "90% CI (MC)". We also show the empirical, as well as the population implied ϕ_n from the MTSM for maturities ranging from $n = 2$ to 40 quarters in this plot.

As can be seen, the estimated MTSM can generate the overall negative value for all $\{\phi_n\}_{n=2}^{40}$ that we consider. However, while the observed $\{\phi_n\}_{n=2}^{40}$ pattern in the empirical estimates shows a decreasing slope, beginning with a small negative estimate of ϕ_2 and increasing to a larger negative estimate of ϕ_{40}, the pattern in the MTSM implied $\{\phi_n\}_{n=2}^{40}$ is reversed in the sense that we start of with a fairly large negative ϕ_2 of around −4.3, which increases with the maturities to around −3 at $n = 40$. This appears to be the largest inconsistency of the estimated MTSM when compared to the empirical data. What is interesting, nevertheless is that from a maturity of about 14 quarters, the confidence intervals (CI) of the empirical and simulated (MC) population model overlap, suggesting that they are not significantly different from one another. From a maturity of about $n = 25$, the match of the empirical and model implied ϕ_n can be seen to be fairly good. That the dynamics of the short rate are not perfectly matched was also reflected in the mild serial correlation of the monetary policy shocks discussed in previous sections.[21] However, compared to the predictions of the expectations hypothesis market by the green line in Fig. 4, our MTSM shows a substantial improvement.

LPY(*ii*): LPY(*ii*) is derived by decomposing the bond return $(y_{t+1}^{(n-1)} - y_t^{(n)})$ into a premium part and an expectations part.[22] After this decomposition, the model-implied premium part is used to *'correct'* the data from the deviations of the expectations hypothesis, yielding:

$$\phi_n^{\text{adj}} = \frac{\text{Cov}\left[(y_{t+1}^{(n-1)} - y_t^{(n)} + \mathcal{D}_{t+1}^{(n)}/(n-1)),\, (y_t^{(n)} - r_t)/(n-1)\right]}{\text{Var}\left[(y_t^{(n)} - r_t)/(n-1)\right]}. \tag{45}$$

The theoretical population value of ϕ_n^{adj} should by definition, be unity. When calculating the model implied $\mathcal{D}_{t+1}^{(n)}$ using actual yields, we can test whether the model implied risk premium corrects for the deviation from the EH. Because of the negative covariance between excess returns and the slope of the yield curve (reflected in the negative regression coefficient in Table 6), $\mathcal{D}_{t+1}^{(n)}$ has to have a positive covariance with the slope, so that LPY(*ii*) is matched. In Panel (b) of Fig. 4 we show the model implied ϕ_n^{adj} again from the population model, as well as from the small sample Monte Carlo simulation that we implement, as well as the $\hat{\phi}_n$ estimated from the sample data (ie., the CS regression coefficients).

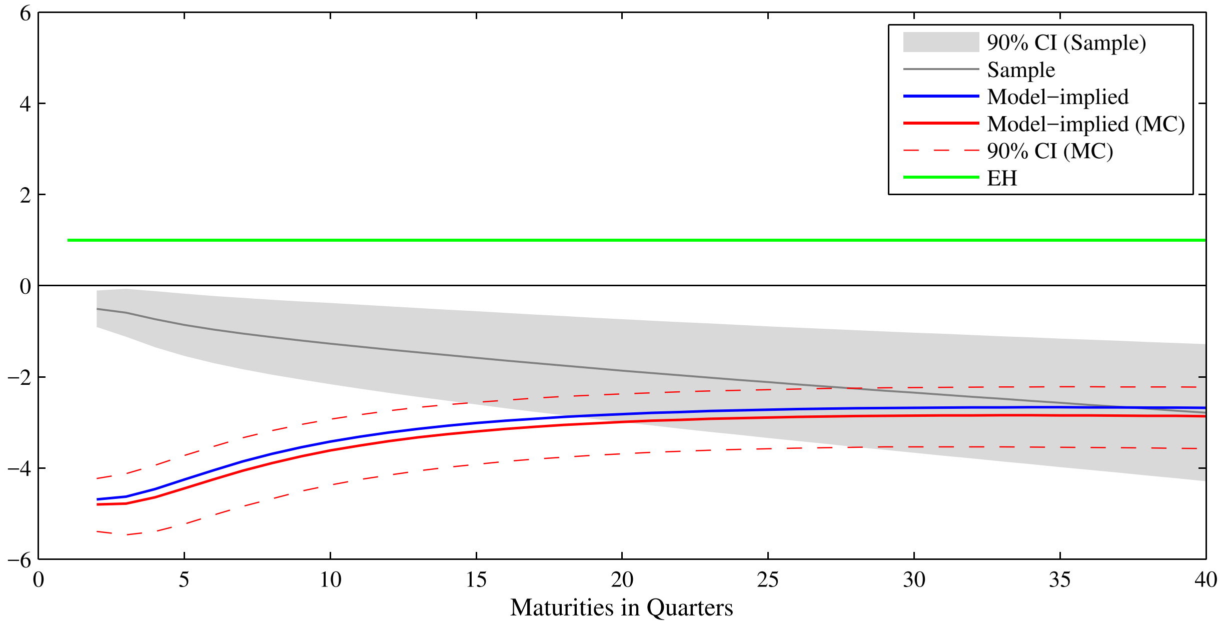
From this plot we can see that our time-varying term premium estimates are able to correct for most of the deviations from the expectations hypothesis. The model implied adjusted regression coefficients ϕ_n^{adj} of LPY(*ii*) are always above the CS regression coefficients. The adjustment term $\mathcal{D}_{t+1}^{(n)}/(n-1)$ appears to work particularly well at long horizon maturities, with the simulation based model-implied ϕ_n^{adj} producing confidence intervals that include the true value of unity at a maturity horizon of $n > 22$.

To summarize the findings of this section, we can state that our model appears to do reasonably well in tests of the expectations hypothesis, despite the fact that we only work with one yield curve factor in addition to the two macroeconomic

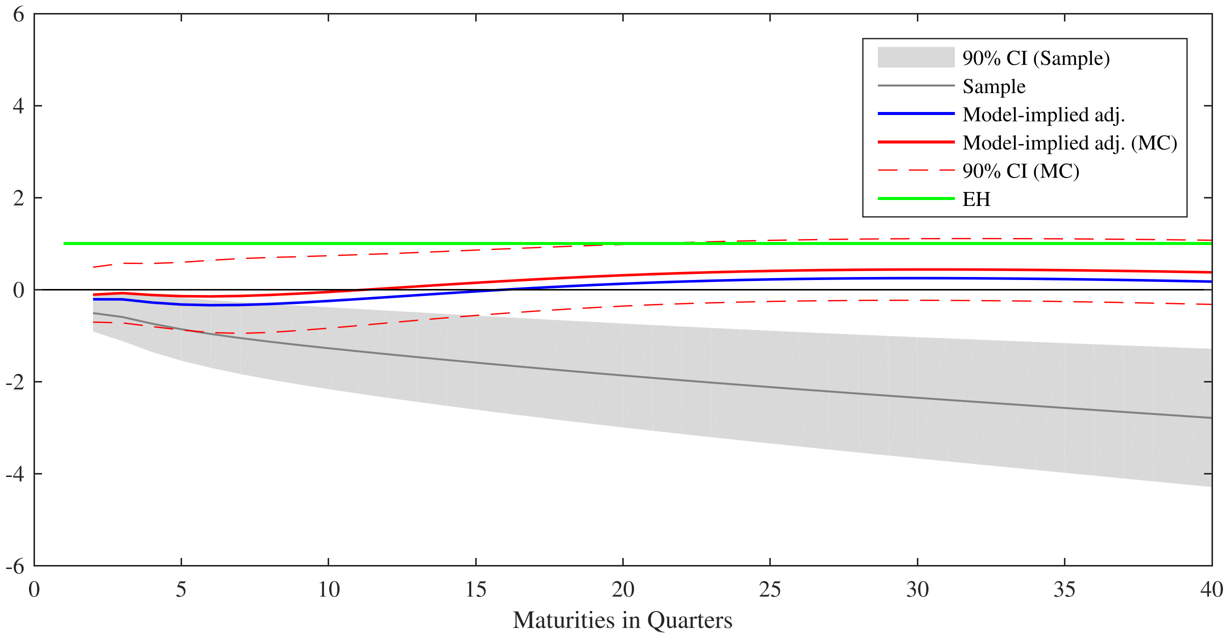
[20] In Appendix A.3 we show how these quantities are calculated in population from the model.

[21] Although it may be possible to better match the historical behaviour of yields under the physical measure by increasing the state vector with additional yield curve factors, we leave this aspect for another study.

[22] Appendix A.4 shows how to do this.



(a) LPY(i)



(b) LPY(ii)

Fig. 4. Model implied regression coefficients LPY(i) and LPY(ii). The top Panel (a) shows estimates of ϕ_n from the regression model: $(y_{t+1}^{(n-1)} - y_t^{(n)}) = \phi_0 + \phi_n[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$ and corresponding model-implied coefficient values. The bottom Panel (b) shows estimates of ϕ_n^{adj} from the regression model: $(y_{t+1}^{(n-1)} - y_t^{(n)} + \mathcal{D}_{t+1}^{(n)}/(n-1)) = \phi_0 + \phi_n^{\text{adj}}[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$ and corresponding model-implied coefficient values. The population coefficients are the theoretical values based on our parameter estimates; the MC coefficients are obtained from a small Monte–Carlo exercise described in the text.

factors. Our LPY(i) and LPY (ii) diagnostic test results of Dai and Singleton (2002) are overall positive, especially at longer maturities. The implied term-premium, a key variable in the analysis of the impulse responses that we presented in the previous section, thus appears to be of a sensible magnitude.[23]

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[23] Appendix A.5 reports estimation results over a shorter sample period that ends in 2006:Q4 and thus excludes the global financial crisis period from the analysis as robustness check of our findings.

5. Conclusion

We estimate a Gaussian macro-finance term structure model that incorporates a small scale New Keynesian policy model and no arbitrage restrictions on the yield curve for the U.S. economy. To alleviate the computational demands of the estimation of our model, we utilize the computationally convenient normalization of Joslin et al. (2013b) to bypasses the need to put any '*arbitrary*' zero restrictions on the parameters driving the market price of risk.

Using quarterly data from 1962:Q1 to 2014:Q2, we find that inflation and the output gap account for about 80% of the total forecast error variance of yields at the short and medium ends of the yield curve, and that monetary policy shocks account for about 20%. Real economic activity explains (at most forecasting horizons) a larger part of the forecast error variance than inflation. Results for term premia yield qualitatively similar results. Our impulse response function analysis shows that positive shocks to inflation and the output gap decrease the term premium instantaneously. Market participants anticipate that the central bank will increase the short rate in response to improving economic conditions and rising inflation. Correspondingly, we find an instantaneous increase in the expectations hypothesis component of interest rates. These two effects offset each other initially. The term premia responses revert back to zero relatively quickly, while the responses of the expectations hypothesis term are longer lasting, reverting back toward zero rather slowly. In line with a standard Taylor type policy rule, the response of bond yields to macroeconomic shocks is slow initially, reaching its peak after about 4 quarters, with inflation shocks increasing bond yields by more than one-to-one and output shocks by less than one-to-one. Our term structure model implied pricing errors are on average around 65 basis points, with pricing errors increasing with maturity. These pricing errors are correlated with the slope of the yield curve, indicating that the inclusion of a factor correlated with the slope could reduce the pricing errors. Our term premium estimate is strongly counter cyclical, with a correlation of −0.83 with real activity, and captures salient features of the term structure of interest rates that represents a puzzle for the expectations hypothesis.

There are two aspects that are of interest for future work. First, one could expand the state vector of the New Keynesian policy model to include also a long term interest rate. This would allow to price the cross section of yields more accurately, while maintaining our framework of observable pricing factors. The inclusion of a long rate would also make it possible to have the long rate influence the output gap, rather then the short rate, which appears to be more realistic given that savings and investment decisions in an economy are commonly based on long term interest rates. Second, it may be useful to allow the short rate to be measured *with* error. Joslin et al. (2013b) have demonstrated that the impulse responses of yields to macroeconomic variables depend on the choice of yields which are allow to be measured with and without error. Adding measurement errors to all yields could be a potentially interesting way to perform another robustness check.

Appendix A

This appendix provides some additional results related to the derivation of the bond price recursions, the exact matrices corresponding to the NKPM model solution, as well as details about the construction of the LPY projection coefficients. Due to the comments from a referee, we also include a separate Section A.5 that repeats the same analysis that we have done in the main part of the paper, but now covering the shorter period from 1962:Q2 to 2006:Q4.[24] To avoid repetition of the discussion, we provide tables and figures as additional information, but do not include a discussion of the results.

A.1. Bond price recursions

To derive the bond price coefficients a_n and b_n, we use the reduced-form pricing kernel process as in Ang and Piazzesi (2003).

$$\begin{aligned} p_t^{(n+1)} &= \mathbb{E}_t\left[\mathcal{M}_{t+1} p_{t+1}^{(n)}\right] && \text{(A.1)} \\ &= \mathbb{E}_t\left[\exp\left\{-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t\boldsymbol{\epsilon}_{t+1} + A_n + \boldsymbol{B}_n\mathbf{X}_{t+1}\right\}\right] \\ &= \exp\left\{-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t + A_n\right\}\mathbb{E}_t\left[\exp\left\{-\boldsymbol{\lambda}_t'\boldsymbol{\epsilon}_{t+1} + \boldsymbol{B}_n'\mathbf{X}_{t+1}\right\}\right] \\ &= \exp\left\{-r_t - \frac{1}{2}\lambda_t'\lambda_t + A_n\right\}\mathbb{E}_t\left[\exp\left\{-\boldsymbol{\lambda}_t'\boldsymbol{\epsilon}_{t+1} + \boldsymbol{B}_n'(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_t)\right\}\right] \\ &= \exp\left\{-\delta_0 + A_n + \boldsymbol{B}_n'\boldsymbol{\mu} + (\boldsymbol{B}_n'\boldsymbol{\Phi} - \delta_1)\mathbf{X}_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t\right\}\mathbb{E}_t\left[\exp\left\{(-\lambda_t' + B_n'\Sigma)\boldsymbol{\epsilon}_{t+1}\right\}\right] \\ &= \exp\left\{-\delta_0 + A_n + \boldsymbol{B}_n'(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0) + \frac{1}{2}\boldsymbol{B}_n'\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t\boldsymbol{B}_n + (-\boldsymbol{\delta}_1 + \boldsymbol{B}_n'\boldsymbol{\Phi} - \boldsymbol{B}_n'\boldsymbol{\Sigma}\boldsymbol{\lambda}_1)\mathbf{X}_t\right\} && \text{(A.2)} \end{aligned}$$

[24] We would like to thank an anonymous referee for pointing this out to us, and that it may be informative to do so.

A.2. Solving the structural macro model

The matrices $\boldsymbol{G}_0$, $\boldsymbol{G}_1$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Pi}$ from (31) are defined as:

$$\boldsymbol{G}_0 = \begin{bmatrix} 1 & 0 & -\alpha & 0 & 0 & -\theta_\pi & \rho_\pi\theta_\pi & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \beta & -\beta & \rho_x\beta & -\theta_x & \rho_x\theta_x \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -(1-\theta_r)\kappa_\pi & 0 & -(1-\theta_r)\kappa_x & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.3}$$

$$\boldsymbol{G}_1 = \begin{bmatrix} (1-\theta_\pi+\rho_\pi) & -\rho_\pi(1-\theta_\pi) & -\rho_\pi\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\theta_x+\rho_x) & -\rho_x(1-\theta_x) & \rho_x\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.4}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \sigma_\pi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_r \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\Pi} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A.5}$$

A.3. Calculating the projection coefficient LPY(i)

Here we derive the projection coefficient in Eq. (44) in population:

$$\begin{aligned} E[(y_{t+1}^{(n-1)} - y_t^{(n)})(y_t^{(n)} - r_t)] = {} & (a_{n-1} - a_n)(a_n - a_1) + (a_{n-1} - a_n)(\boldsymbol{b}_n - \boldsymbol{b}_1)'\mathbb{E}[\mathbf{X}_t] \\ & + (a_n - a_1)\mathbb{E}[(\boldsymbol{b}_{n-1}'\mathbf{X}_{t+1} - \boldsymbol{b}_n'\mathbf{X}_t)] + \boldsymbol{b}_{n-1}'\mathbb{E}[(\mathbf{X}_{t+1}\mathbf{X}_t')](\boldsymbol{b}_n - \boldsymbol{b}_1) - \boldsymbol{b}_n'\mathbb{E}[(\mathbf{X}_{t+1}\mathbf{X}_t')](\boldsymbol{b}_n - \boldsymbol{b}_1). \end{aligned}$$

The variance of $\mathbf{X}_t$ is calculated use standard matrix formulae as:

$$\begin{aligned} \mathbf{X}_t &= \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{X}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_t, \\ \text{Var}(\mathbf{X}_t) &= \boldsymbol{\Phi}\text{Var}(\mathbf{X}_{t-1})\boldsymbol{\Phi}' + \boldsymbol{\Sigma}\boldsymbol{\Sigma}', \\ \text{vec}[\text{Var}(\mathbf{X}_t)] &= (\boldsymbol{\Phi} \otimes \boldsymbol{\Phi})\text{vec}[\text{Var}(\mathbf{X}_{t-1})] + \text{vec}[\boldsymbol{\Sigma}\boldsymbol{\Sigma}'], \\ &= (\boldsymbol{I}_k - \boldsymbol{\Phi} \otimes \boldsymbol{\Phi})^{-1}\text{vec}[\boldsymbol{\Sigma}\boldsymbol{\Sigma}'], \end{aligned} \tag{A.6}$$

where vec[·] denotes the stacking operator and $\otimes$ the Kroenecker product, and due to the state vector being stationary, we have that $\text{Var}(\mathbf{X}_t) = \text{Var}(\mathbf{X}_{t-1})$.

A.4. Calculating the projection coefficient LPY(ii)

Starting from the definition of a realized excess one-period return ($\tilde{\mathcal{D}}_{t+1}^n$), we can derive LPY($ii$).

$$\begin{aligned} \tilde{\mathcal{D}}_{t+1}^{(n)} &\equiv n y_t^{(n)} - (n-1)y_{t+1}^{(n-1)} - r_t, \\ &= (n-1)(y_t^{(n)} - y_{t+1}^{(n-1)}) + y_t^{(n)} - r_t. \end{aligned}$$

Applying the time t expectations operator and rearranging yields

$$\mathbb{E}_t[y_{t+1}^{(n-1)} - y_t^{(n)} + \frac{1}{n-1}\tilde{D}_{t+1}^{(n)}] = \frac{1}{n-1}(y_t^n - r_t^1) \tag{A.7}$$

Now, by linking $E_t[\tilde{\mathcal{D}}^{(n)}_{t+1}]$ to the risk premiums of an economic model adds economic content to Eq. (A.7). Decompose the excess return $\tilde{\mathcal{D}}^{n}_{t+1}$ into a premium part and an expectations part and note that the expectations part $\mathbb{E}_t r_{t+i} - \mathbb{E}_{t+1} r_{t+i}$ has a time t conditional expectation of zero, we obtain:

$$\tilde{\mathcal{D}}^{(n)}_{t+1} = \mathcal{D}^{(n)}_{t+1} + \sum_{i=1}^{n-1} (\mathbb{E}_t r_{t+i} - \mathbb{E}_{t+1} r_{t+i}),$$

where

$$\begin{aligned}\mathbb{E}_t[\tilde{\mathcal{D}}^{(n)}_{t+1}] &= \mathbb{E}_t[\mathcal{D}^{(n)}_{t+1}] \\ &= -(n-1)\mathbb{E}_t[(yrp^{(n-1)}_{t+1} - yrp^{(n-1)}_t)] + p^{(n-1)}_t,\end{aligned}$$

with $yrp^{(n)}_t$ the yield risk premium, the average expected return from holding a n-period bond to maturity, financed by a sequence of one period bonds, is. In light of this, we can replace $\mathbb{E}_t[\tilde{\mathcal{D}}^{(n)}_{t+1}]$ in Eq. (A.7) with $\mathbb{E}_t[\mathcal{D}^{(n)}_{t+1}]$ and formulating a regression model with the regression coefficient:

$$\phi^{\text{adj}}_n = \frac{\text{Cov}[(y^{(n-1)}_{t+1} - y^{(n)}_t + \mathcal{D}^{(n)}_{t+1}/(n-1)), (y^{(n)}_t - r_t)/(n-1)]}{\text{Var}[(y^{(n)}_t - r_t)/(n-1)]}, \tag{A.8}$$

where $y^{(n)}_{t+1}$ are actual yield data and $\mathcal{D}^{(n)}_{t+1}$ is calculated from the model implied risk premiums. (Figs. A.1 A.2 and A.3).

A.5. Robustness check: results for the period up to 2006:Q2

We present additional estimation results from our structural MTSM model outlined in Section 2 in the tables and figures that are reported below, now over the shorter period from 1962:Q2 to 2006:Q4. The tables and figures have the same structure as in the main text to facilitate a direct comparison. (Tables A.1 A.2 A.3 A.4 A.5 and A.6)

Table A.1
Maximum likelihood estimates of the NKPM structural parameters 1962:Q2 to 2006Q4.

Equation	Parameter	Estimate	Standard error
Inflation dynamics:	θ_π	0.4444***	0.0830
$\pi_t = \theta_\pi \mathbb{E}_t[\pi_{t+1}] + (1-\theta_\pi)\pi_{t-1} + \alpha x_t + \varepsilon^\pi_t$	α	0.0108	0.0106
$\varepsilon^\pi_t = \rho_\pi \varepsilon^\pi_{t-1} + \nu^\pi_t,$	ρ_π	0.5118***	0.0696
Output dynamics	θ_x	0.5195***	0.0122
$x_t = \theta_x \mathbb{E}_t[x_{t+1}] + (1-\theta_x)x_{t-1} - \beta(r_t - \mathbb{E}_t[\pi_{t+1}]) + \varepsilon^x_t$	β	−0.0033	0.0025
$\varepsilon^x_t = \rho_x \varepsilon^x_{t-1} + \nu^x_t,$	ρ_x	0.2974***	0.0838
Short rate dynamics	θ_r	0.9099***	0.0287
$r_t = \theta_r r_{t-1} + (1-\theta_r)(\kappa_\pi \mathbb{E}_t[\pi_{t+1}] + \kappa_x x_t) + \varepsilon^r_t$	κ_π	1.6050***	0.3668
	κ_x	0.9990**	0.4519
Standard deviation of $\nu^\pi_t, \nu^x_t, \varepsilon^r_t$ (× 400)	σ_π	0.1689	0.0246
	σ_x	0.4047	0.0228
	σ_r	0.8120	0.0432

This table reports Maximum Likelihood estimates of the structural parameters of the New Keynesian Policy Model (NKPM) as defined in (29). The first column shows the estimating equations, the second column the parameter of interest, the third the estimates and the forth the standard error. The standard errors are based on numerical evaluation of the Hessian matrix. Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

Table A.2
Maximum likelihood estimates of $\mathbb{Q}$ parameters 1962:Q2 to 2006Q4.

Parameter	Estimate	Standard errors
$\lambda^{\mathbb{Q}}_1$	0.9999***	0.0011
$\lambda^{\mathbb{Q}}_2$	0.8955***	0.0048
$\lambda^{\mathbb{Q}}_3$	0.2789***	0.0109
$\sigma_u(\times 400)$	0.5627***	0.0020

This table reports Maximum Likelihood estimates of the $\mathbb{Q}$ parameters ($\mathbf{\Psi}^{\mathbb{Q}}$) as defined in (22). Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

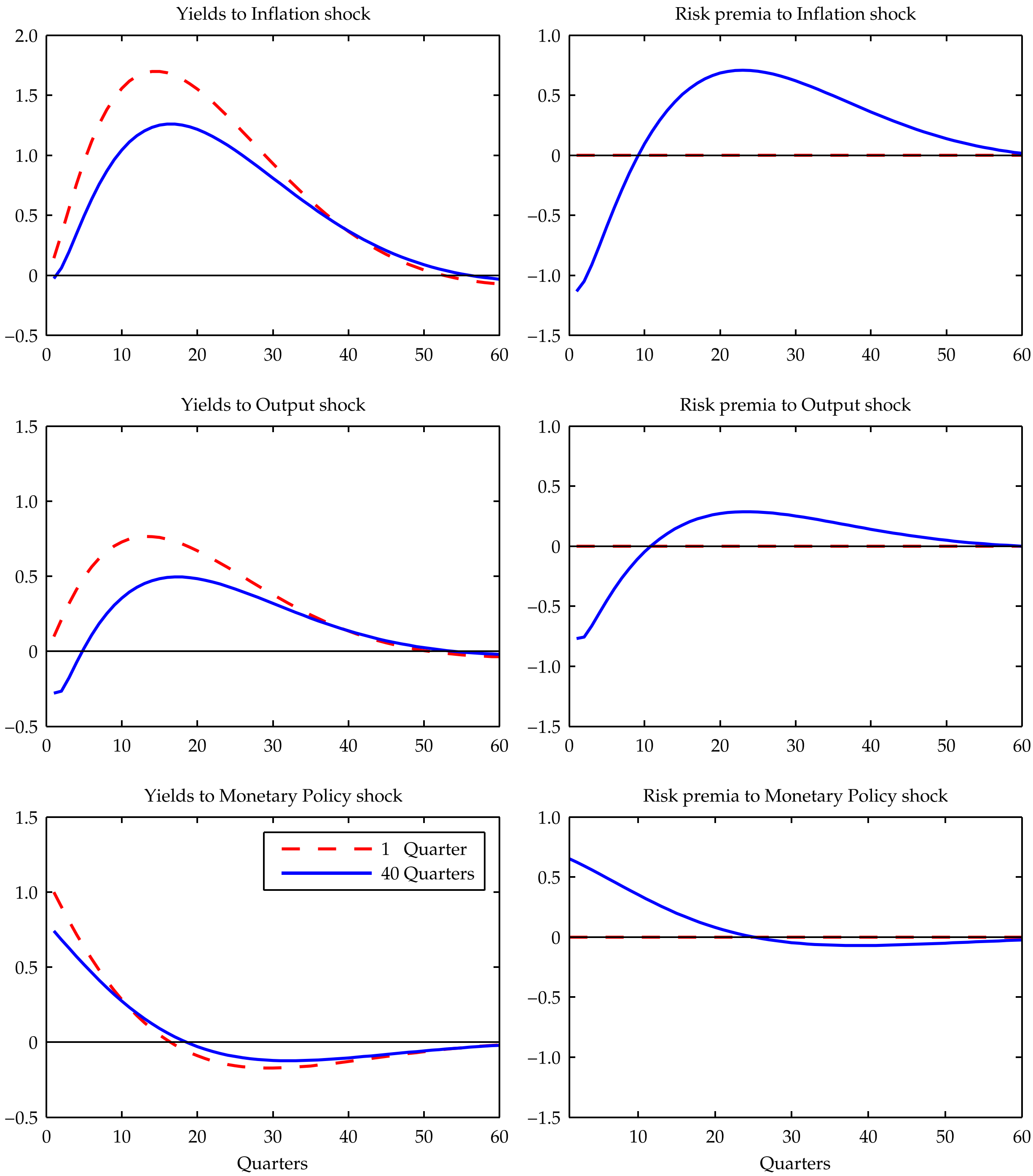


Fig. A.1. Impulse response functions of yields and risk premia at the 1 quarter and 40 quarter horizon to shocks in inflation, output, and monetary policy. Shocks are normalized to one-unit shocks.

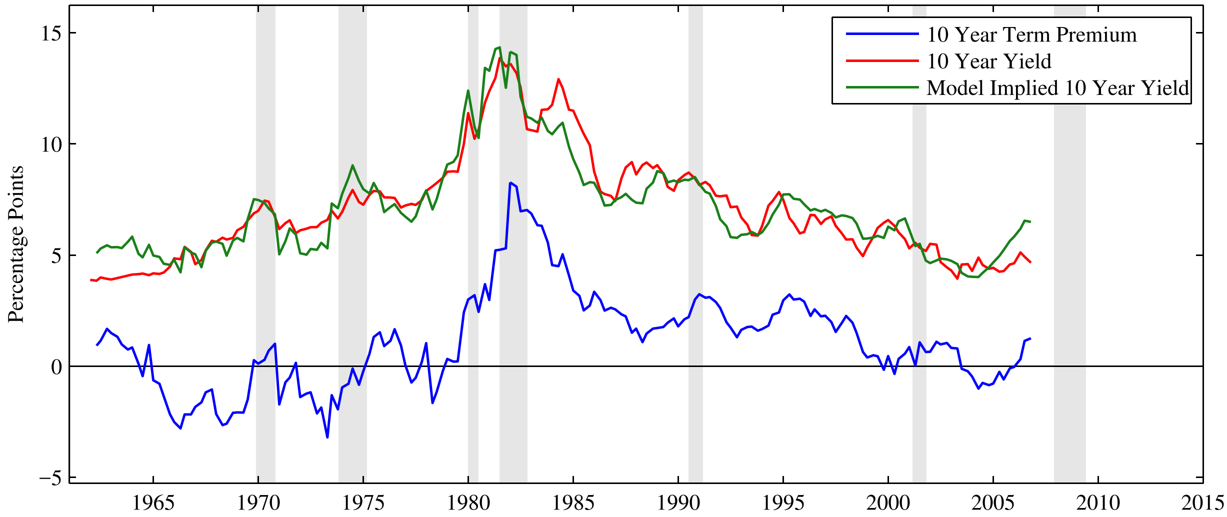


Fig. A.2. Term premium and bond yields: This figure shows the term premium component of the ten year spot rate as estimated from our MTSM described in the text (blue line), together with the model implied 10 Year Yield (green line) and the (empirical) 10 Year Yield ($y_t^{(40)}$) (red line) from the Gürkaynak et al. (2007) database. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table A.3
Maximum likelihood estimates of macro spanning parameters 1962:Q2 to 2006Q4.

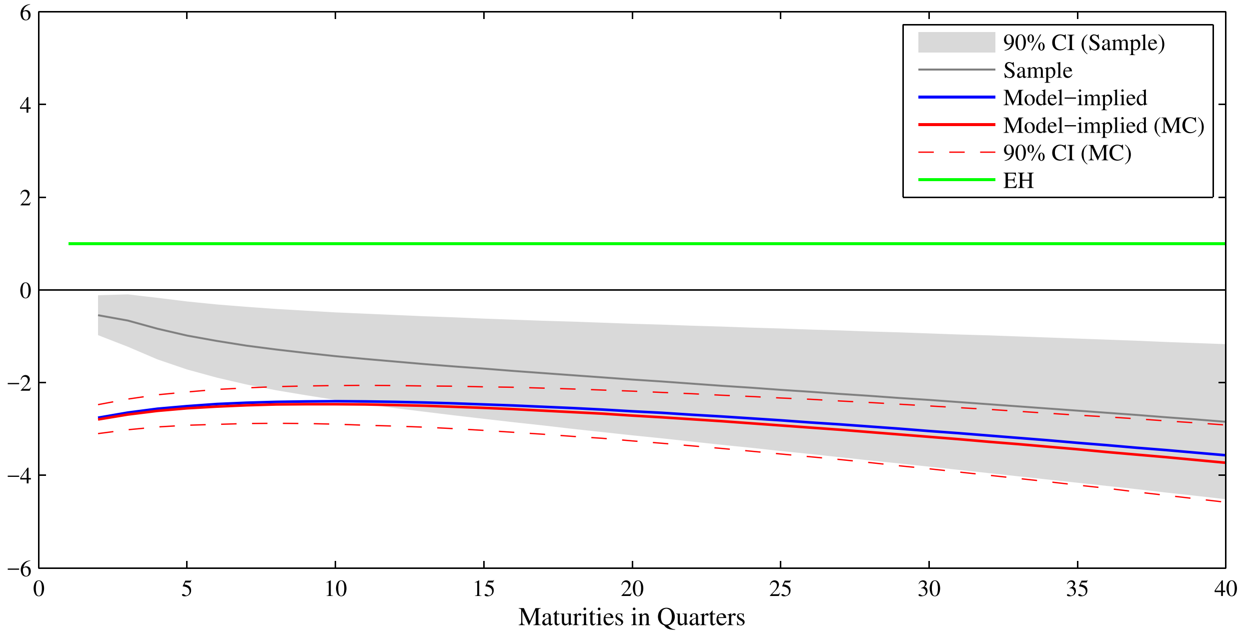
Parameter	$y_t^{(1)}$	$y_t^{(4)}$	$y_t^{(20)}$
π_t	48.09***	−69.74***	24.29***
(std.error)	(2.96)	(4.73)	(1.89)
y_t	−17.59***	29.83***	−14.08***
(std.error)	(2.46)	(3.55)	(1.26)

This table reports the Maximum Likelihood estimates of the macro spanning parameters $\boldsymbol{\gamma}_1$ in (28). Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

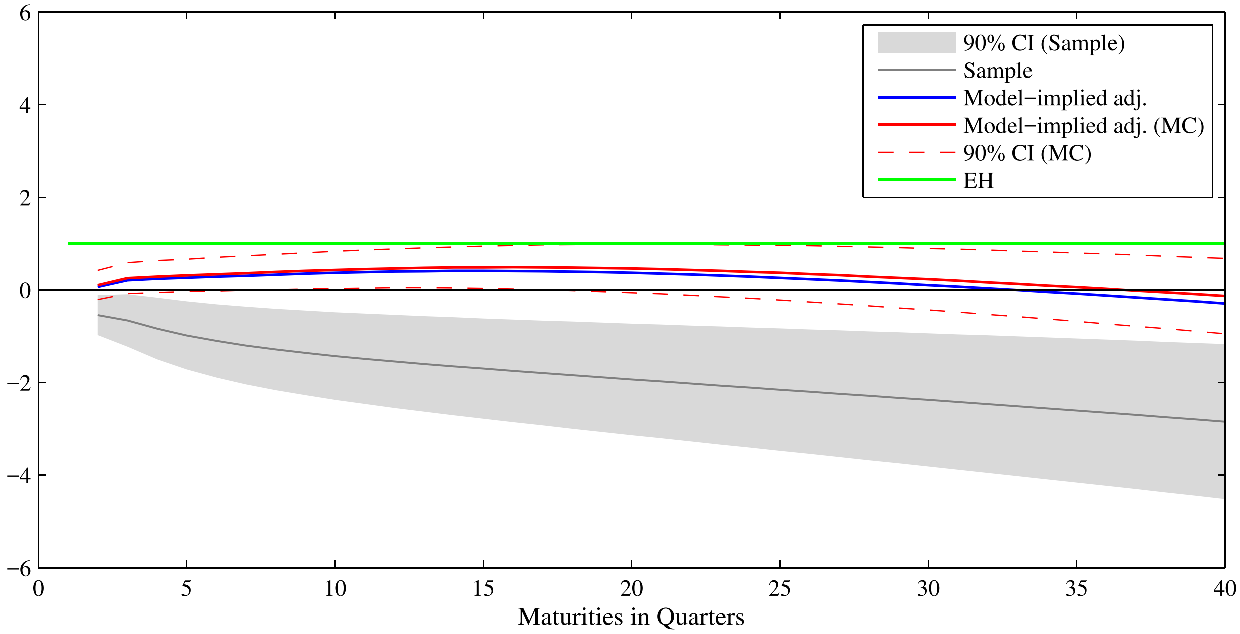
Table A.4
Comparison of implied theoretical and empirical moments of the data 1962:Q2 to 2006Q4.

Moments	Sample				Theoretical			
	π_t	x_t	r_t	$y_t^{(40)}$	π_t	x_t	r_t	$y_t^{(40)}$
σ	2.2537	2.5308	2.8735	2.4894	2.7139	2.3333	3.8848	3.0589
ACF(1)	0.9855	0.9498	0.9542	0.9773	0.9903	0.9406	0.9743	0.9711
ACF(2)	0.9540	0.8740	0.8941	0.9449	0.9697	0.8512	0.9450	0.9380
ACF(3)	0.9110	0.7772	0.8475	0.9154	0.9427	0.7582	0.9127	0.9021
ACF(4)	0.8599	0.6755	0.7894	0.8790	0.9116	0.6699	0.8776	0.8638
Corr(x_t, π_t)	−0.2465				−0.1249			
Corr(x_t, r_t)	0.6889				0.7739			
Corr(r_t, π_t)	−0.1810				−0.1558			

This table shows the comparison between the model implied theoretical moments and the empirical (sample) moments of the data. The unconditional standard deviation is denoted by σ, ACF(·) denotes the value of the autocorrelation function and Corr is the standard (Pearson) correlation.



(a) LPY(i)



(b) LPY(i)

Fig. A.3. Model implied regression coefficients LPY(i) and LPY(ii). The top Panel (a) shows estimates of ϕ_n from the regression model: $(y_{t+1}^{(n-1)} - y_t^{(n)}) = \phi_0 + \phi_n[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$ and corresponding model-implied coefficient values. The bottom Panel (b) shows estimates of ϕ_n^{adj} from the regression model: $(y_{t+1}^{(n-1)} - y_t^{(n)} + \mathcal{D}_{t+1}^{(n)}/(n-1)) = \phi_0 + \phi_n^{\text{adj}}[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$ and corresponding model-implied coefficient values. The population coefficients are the theoretical values based on our parameter estimates; the MC coefficients are obtained from a small Monte–Carlo exercise described in the text.

Table A.5
Variance decompositions.

Short end	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0028	0.0418	0.3196	0.3525
Output	0.0079	0.0809	0.4155	0.4287
Short rate	0.9893	0.8773	0.2649	0.2187
Middle	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0000	0.0176	0.3366	0.3790
Output	0.0477	0.0280	0.3183	0.3635
Short rate	0.9523	0.9544	0.3450	0.2574
Long end	Forecast horizon h (in Quarters)			
	1	4	20	∞
Inflation	0.0002	0.0117	0.3379	0.3851
Output	0.1076	0.0826	0.2966	0.3503
Short rate	0.8923	0.9057	0.3655	0.2646

This table shows variance decompositions, that is, the contribution of each factor to the h-step ahead forecast error variance of the short end, middle end and long end of the yield curve. Each numerical entry in the three separate (3×4) blocks gives the percentage value of the $h-$step ahead forecast error variance explained by inflation, output, and the short rate. For instance, in the Short end block, 99% of the $1-$step ahead forecast error variance at the short end of the yield curve is explained by the short rate itself.

Table A.6
Campbell and Shiller regressions.

Maturity in quarters	Estimate	Standard errors
4	−0.5853	0.4031
12	−1.3227*	0.6096
20	−1.6836**	0.7311
32	−2.1666**	0.9021
40	−2.5382***	1.0162

This table reports the empirical estimates of Campbell and Shiller regression coefficient ϕ_n from the regression $(y_{t+1}^{(n-1)} - y_t^{(n)}) = \phi_0 + \phi_n[(y_t^n - r_t)/(n-1)] + \epsilon_{t+1}$. This regression was estimated with Generalized Least Squares (GLS), where the residuals follow first order autoregressive processes. Significance at the 1, 5, and 10 percent levels are marked, respectively, by ***, ** and * (two sided null hypothesis).

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